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# History, Expectations, and the Geography of Adjustment

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# Bigger Agenda: What Can a Spatial Model Do?

- **Quantitative spatial models** (QSMs) are now a common tool for spatial economists (Redding & Rossi-Hansberg, 2017): tractable GE, fitted to data, used for counterfactuals.
- Understanding a model's **internal working** is essential for interpreting numerical results.
  - Which spatial patterns can it generate? Which does it *stabilize*?
  - How does it *adjust* to a change, and to a change *known in advance*?
- Fitting an observed population distribution well need not establish that a model contains the **endogenous forces** a counterfactual experiment relies on.

This talk explores a dynamic (overlapping-generations) spatial model
in symmetric two- and four-region economies to understand its endogenous forces.

*But how much do endogenous forces matter? Can they change the sign of a prediction?*

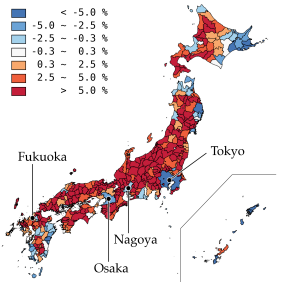
# Same Shock, Qualitatively Opposite Predictions

A lower transport cost can work in two opposite directions. (Akamatsu *et al.*, 2026)

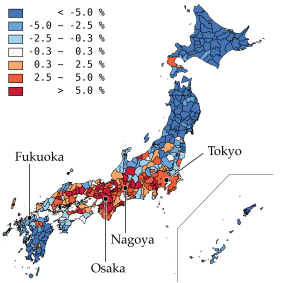
(1) It can weaken the relative advantage of being big ⇒ activity can *disperse*.

(2) It can expand the market reach of big hubs ⇒ activity can *concentrate* further.

Which force wins can flip the **sign** of a counterfactual! (Sugimoto *et al.*, 2025)




(1) > (2): Revival of the **periphery**




(1) < (2): More concentration in the **core**

# Two Fundamental Types of Dispersion Forces

(1) **"Local" dispersion force**

= Crowding *within* a region such as land rents or traffic congestion (Helpman, 1998).
Locally spreads residents **around an existing center**.

(2) **"Global" dispersion force**

= Competition *across* places in the presence of immobile factors (Krugman, 1991b).
Keeps **several economic centers apart** (cf. central place theory).

Thus, the spatial reach of dispersion forces shapes the **number, spacing, and extent** of agglomerations in many conventional static models (Akamatsu *et al.*, 2026).

The AMOT taxonomy of static models

| **Type L** (local) | **Type G** (global) | **Type LG** (local + global) |
|---|---|---|
| Helpman (1998), | Krugman (1991b), | Tabuchi (1998), |
| Redding & Sturm (2008), | Puga (1999), | Kucheryavyy *et al.* (2024), |
| Allen & Arkolakis (2014) | Harris & Wilson (1978) | Sugimoto *et al.* (2025) |

# Added Dimensions in Dynamic Spatial Models

In a dynamic economy, regional advantages also pass **between generations**:

- Migration frictions preserve part of the inherited settlement (**inherited origins**)
- Past settlement affects today's productivity and amenities (**historical spillovers**)
- Households weigh their **descendants'** opportunities (patience)
- A change can be known before it arrives (**advance notice**)

**Questions.**

(i) How do these linkages interact with the spatial reach of dispersion forces to determine *where people move*, and what *spatial patterns* emerge?

(ii) What must we *observe* to discipline a model's prediction of that adjustment?

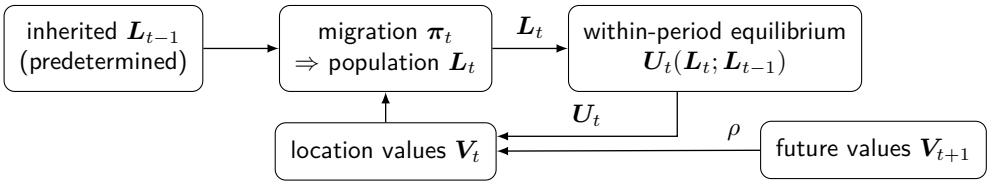
# This Talk: Three Answers

Economy: Kucheryavyy *et al.* (2024) (KLR) goods markets + Allen & Donaldson (2020, 2022) (AD) spillovers and dynastic location choice, taken apart in stylized settings.

1. Patience and earlier notice act on *different margins*:
   - patience moves thresholds and favors one center, and
   - notice moves no threshold and can favor two separated centers
2. The *same* announced improvement makes a region a winner under one dispersion mechanism and a loser under the other.
   - A dynamic analogue of the qualitative reversal in AMOT.
3. A model that fits every local population path after a permanent surprise need not predict adjustment to an *announced* change, even in sign.
   - "News ratios" and changes of known duration supply what is missing.

# Environment and Timing

- $N \geq 2$ regions; period $t$ is **one generation** ($\approx$ 25–30 years; a decade in AD).
- Each generation: a unit mass of mobile manufacturing workers, $\boldsymbol{L}_t = (L_{it})_{i=1}^N$. Region $i$ may host a fixed mass $\bar{L}_i^{\mathrm{A}}$ of **immobile** agricultural workers.
- Each worker has one descendant, who **inherits the parent's location as origin**.
- Migration choice weighs the next generation's opportunities. (Aggregate paths are perfectly foreseen given the information available, as idiosyncratic tastes average out.)



# Within-Period: KLR Goods Markets + AD Spillovers

- One Armington variety per region. Unit cost $c_{it} = w_{it}^{\zeta} P_{it}^{1-\zeta} / a_{it}$ with intermediates, labor share $\zeta$. Trade elasticity $\varepsilon$, iceberg trade costs $\tau_{ij}$.
- Spending: share $\gamma$ on manufactures, $1-\gamma$ on freely traded agriculture (numeraire).
- Period utility and AD-style spillovers, with current and historical elasticities:

$$U_{it} = b_{it} \frac{w_{it}}{P_{it}^{\gamma} \left(w_t^{\mathrm{A}}\right)^{1-\gamma}} \qquad a_{it} = \bar{a}_i \, L_{it}^{\alpha_1} L_{i,t-1}^{\alpha_2} \qquad b_{it} = \bar{b}_i \, L_{it}^{\beta_1} L_{i,t-1}^{\beta_2} \, e^{\omega_{it}}$$

  - $\omega_{it}$ is an exogenous log-amenity change (the "announced improvement" later).

**Type L** economy: $\gamma = 1$, all demand moves with workers (Helpman, 1998; Allen & Arkolakis, 2014).

**Type LG** economy: $\gamma < \min\{1, 2\varepsilon\}$, immobile agricultural demand anchors part of manufacturing demand to fixed locations (Krugman, 1991b; Kucheryavyy *et al.*, 2024).

# Migration and Dynastic Location Values

- Migration cost $\mu_{ij} \geq 1$ ($\mu_{ii} = 1$); Fréchet location tastes with shape $\theta$:

$$L_{jt} = \sum_i \pi_{ij,t}\, L_{i,t-1}, \qquad \text{where} \qquad \pi_{ij,t} = \frac{\mu_{ij}^{-\theta} V_{jt}^{\theta}}{\sum_h \mu_{ih}^{-\theta} V_{ht}^{\theta}}.$$

- Location value with *dynastic weight* $\rho \in [0, 1)$:

$$V_{it} = U_{it}(\boldsymbol{L}_t; \boldsymbol{L}_{t-1}) \Big( \sum_j \mu_{ij}^{-\theta} V_{j,t+1}^{\theta} \Big)^{\rho/\theta}.$$

  - $\rho = 0$: myopic location choice. With $\rho > 0$, a location is attractive partly for the opportunities it offers a descendant who starts there.

- **Dynamic equilibrium**: positive population and value paths $\{(\boldsymbol{L}_t, \boldsymbol{V}_t)\}_{t=0}^{\infty}$ satisfying market clearing, migration, and the value recursion from a given initial distribution (no bubbles).

  - **Steady state**: dynamic equilibrium with constant population and values.

# Intergenerational Linkages

| Linkage | Model primitives | Effective object in the dynamics | |
|---|---|---|---|
| (1) Inherited origins | $\mu, \theta$ | population inertia | $\psi$ |
| (2) Historical spillovers | $\alpha_2, \beta_2$ | historical utility effect | $q$ |
| (3) Descendants' opportunities | $\rho$ | continuation weight | $\rho\psi$ |
| (4) Information | $H$ (notice horizon) | accumulation before $T$ | |

Even without (2) and (3), migration frictions link successive distributions.

**Claim** **The four mechanisms act on *space* differently.**

The footers on later slides highlight the mechanisms at work.

# Selected Literature [if time]

- **Endogenous forces and geography.** AMOT organize theories of spatial agglomeration, e.g., Krugman (1991b); Helpman (1998), by the spatial reach of dispersion forces. Spectral methods for symmetric geographies: Akamatsu *et al.* (2012); Ikeda & Murota (2014).
- **History vs expectations.** Krugman (1991a); Matsuyama (1991); Fukao & Bénabou (1993); selection with a potential, Oyama (2009); a quantitative study of Hiroshima, Takeda & Yamagishi (2026).
- **Dynamic QSMs.** Allen & Donaldson (2020, 2022): persistence, path dependence, and the dynastic structure used here. Kleinman *et al.* (2023): spectral linearization with forward-looking migration. The two-region system here has explicit roots.
- **Validation.** Adão *et al.* (2025): test a model's causal prediction for an observed change.

# Population Changes and Two Eigenvalues

- Two identical regions $\Rightarrow$ the uniform distribution $L_1 = L_2 = \frac{1}{2}$ is a steady state. Define **relative population** $\ell_t = \log(L_{1t}/L_{2t})$.
- Near symmetry, spatial interaction with *freeness* $\phi \in (0, 1)$ *attenuates* a population advantage $\ell > 0$ by $\frac{1-\phi}{1+\phi}$.
  - Generic accessibility $A_i = L_i + \phi L_j \Rightarrow$ a shift $\ell$ moves $\log(A_1/A_2)$ by only $\frac{1-\phi}{1+\phi}\,\ell$.
- Trade and migration each supply one such number, both rising with their cost:

$$\chi = \frac{1 - \tau^{-\varepsilon}}{1 + \tau^{-\varepsilon}} \qquad \text{and} \qquad \psi = \frac{1 - \mu^{-\theta}}{1 + \mu^{-\theta}}.$$

  - $\chi$: how much of a population advantage survives in goods-market access.
  - $\psi$: **population inertia**. At equal location values, the fraction $\psi$ of an inherited population difference survives into the next generation.

# Linearized Dynamics Around Symmetry

With $u_t = \log(U_{1t}/U_{2t})$, $v_t = \log(V_{1t}/V_{2t})$, $\omega_t = \omega_{1t} - \omega_{2t}$, around $\boldsymbol{L} = (1/2, 1/2)$,

$$
\begin{aligned}
u_t &= p\,\ell_t + q\,\ell_{t-1} + \omega_t && \text{(period utility)} \\
\ell_t &= \psi\,\ell_{t-1} + m\,v_t && \text{(migration)} \\
v_t &= u_t + \rho\psi\,v_{t+1} && \text{(descendants' opportunities)}
\end{aligned}
$$

- $p$ and $q$: effects of current and inherited population on period utility, respectively:

$$p = (\zeta + \alpha_1)\,\mathcal{M}(\chi) - 1 + \beta_1, \qquad q = \alpha_2\,\mathcal{M}(\chi) + \beta_2, \qquad m = \theta(1 - \psi^2).$$

- $\mathcal{M}(\chi) > 0$: the **real-income response coefficient** in $\mathrm{KLR}$ goods-market equilibrium.

# Local Population Dynamics

For $0 < \rho, \psi < 1$, eliminating location values gives:

$$\rho\psi\,\ell_{t+1} - (1 - mp + \rho\psi^2)\,\ell_t + (\psi + mq)\,\ell_{t-1} = -m\,\omega_t.$$

A path proportional to $\lambda^t$ solves $Q(\lambda) = \rho\psi\lambda^2 - (1 - mp + \rho\psi^2)\lambda + \psi + mq = 0$.

A difference that persists unchanged ($\lambda = 1$) **generates** $u = G^+\ell$ and **requires** $u = \Theta^+\ell$:

$$G^+ = p + q = (\zeta + \alpha_1 + \alpha_2)\,\mathcal{M}(\chi) - 1 + \beta_1 + \beta_2, \qquad \Theta^+ = \frac{1 - \rho\psi}{\theta(1 + \psi)}.$$

- *Generated*: the utility advantage of a lasting concentration.
- *Required*: what migration frictions and forward-looking choice need to keep it in place.

  They balance each other when $Q(1) = m(G^+ - \Theta^+) = 0$.

# Stability: Positive Saddle vs Source

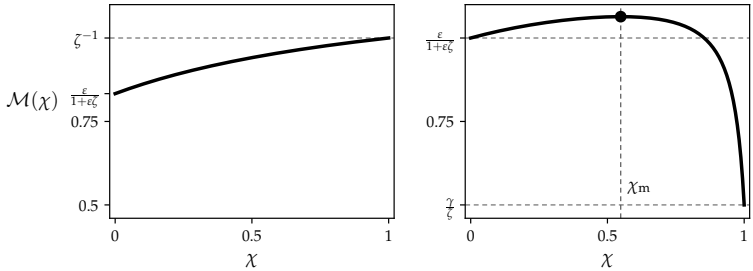
Suppose historical utility effects are nonnegative, $q \geq 0$.

- $G^+ < \Theta^+$: **positive saddle**, $0 < \lambda_s < 1 < \lambda_u$. Every nearby initial distribution has a unique path converging to symmetry: *stable*.
- Crossing $G^+ = \Theta^+$ from below at fixed $q$ makes symmetry a **source**: no nonconstant local path converges. This *instability* lasts until any later crossing at $\lambda = -1$.
- Migration frictions and patience both *lower the required advantage*:

$$\Theta^+(\rho) = \Theta^+(0) - \frac{\rho\psi}{\theta(1+\psi)}, \qquad \Theta^+(0) = \frac{1}{\theta(1+\psi)}.$$

  For a lasting advantage, $v = u/(1-\rho\psi)$: settling where utility is higher also gives descendants better access, so a smaller current advantage sustains the same concentration. Patience has no effect at $\psi = 0$.

# Trade Costs: Enter Only Through $\mathcal{M}(\chi)$ [if time]




(a) Type L economy (b) Type LG economy

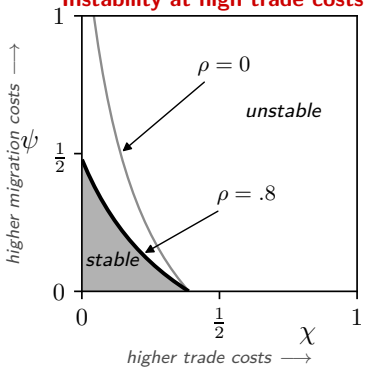
- **Type L**: $\mathcal{M}$ increases in $\chi$. Concentration worsens a region's terms of trade, most at low trade costs: a *local* dispersion force that strengthens as trade costs fall (Helpman, 1998).
- **Type LG**: $\mathcal{M}$ is single-peaked. At high trade costs, serving immobile consumers from a distant center is costly: a *global* dispersion force (Krugman, 1991b).

Panel (a): $(\varepsilon, \zeta, \gamma) = (5, 1, 1)$. Panel (b): $(5, .8, .4)$. Higher $\chi$ means higher trade costs.

# Stability of Symmetry in the $(\chi, \psi)$ Plane ($G^+ < \Theta^+$)

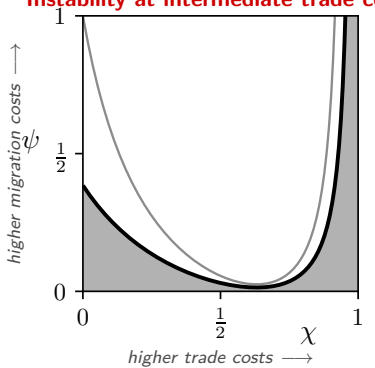
**Type L** $(\gamma = 1)$ $G^+$ rises with $\chi$

**Instability at high trade costs.**




**Type LG** $(\gamma = .5)$ $G^+$ is single-peaked

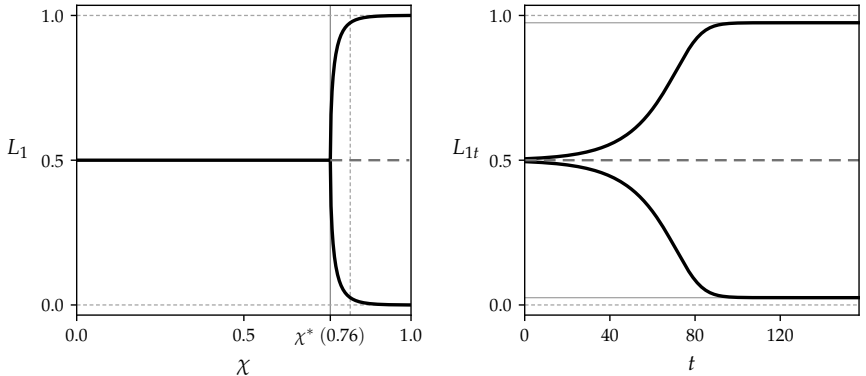
**Instability at intermediate trade costs.**




Migration frictions and patience enter only $\Theta^+$: the stable region shrinks from gray to black.

Both panels: $(\theta, \alpha_1, \alpha_2, \beta_1, \beta_2) = (5, .20, .10, -.30, .30)$, so $q > 0$; Type L $(\varepsilon, \zeta, \gamma) = (5, 1, 1)$, Type LG $(5, .8, .5)$.

# Endogenous Agglomeration in Type L When $G^+ > \Theta^+$




(a) Steady-state populations

(b) Nonlinear adjustment

- History matters in this example: the initially larger region keeps attracting population.

$\varepsilon = \zeta = \gamma = 1$, $\alpha_1 = \alpha_2 = 0$, $(\beta_1, \beta_2) = (.12, .03)$, $(\mu, \theta, \rho) = (2, 4, .6)$; panel (b) at $\chi \approx .82$.

# Recovery and Anticipation

Around a stable symmetric steady state, the response to a known amenity path is unique and bounded: inherited population is predetermined and location values jump (Blanchard & Kahn, 1980).

$$z_t \equiv \ell_t - \lambda_s \ell_{t-1} \qquad \Longrightarrow \qquad z_t = \frac{m}{\rho\psi} \sum_{j \geq 0} r^{\,j+1}\, \omega_{t+j}, \qquad r \equiv \lambda_u^{-1} \in (0,1).$$

- $z_t$: movement **newly induced** at $t$ beyond what propagates at the stable rate. Population responds today to the *entire known future*, discounted at the **anticipation factor** $r$.
- A one-period improvement at $T$, known at $t \leq T$, contributes $m\omega r^{T-t+1}/(\rho\psi)$: one more generation of distance multiplies by $r$. Once it has passed, population recovers at $\lambda_s$.
- $\lambda_s$ = equilibrium persistence, $r$ = the anticipation factor.

# Same Steady States, Different Transitions [if time]

At a steady state, spillovers enter only through $\alpha_1+\alpha_2$ and $\beta_1+\beta_2$ (e.g., $L_{it}^{\alpha_1} L_{i,t-1}^{\alpha_2} = L_i^{\alpha_1+\alpha_2}$).

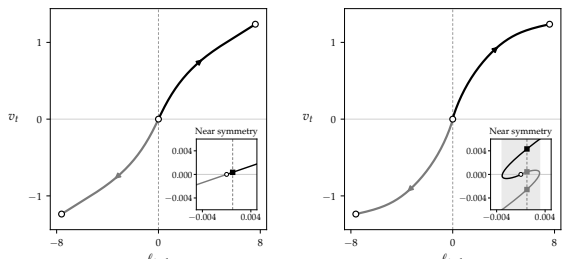
Moving spillovers from current to inherited population ($p \to p-h$, $q \to q+h$) leaves every steady state and the long-run multipliers unchanged, but not the roots:

| | steady states | recovery $\lambda_s$ | anticipation $r$ |
|---|---|---|---|
| spillovers shifted to inherited population ($h \uparrow$) | unchanged | $\uparrow$ (slower) | $\downarrow$ (less) |
| greater patience ($\rho \uparrow$; $p, q, \theta, \psi$ fixed) | threshold moves | weakly $\uparrow$ | $\uparrow$ (more) |

- Historical effects and patience can produce similar recovery, while moving *anticipation in opposite directions*.
- The validation problem to come: permanent-surprise paths reveal $\lambda_s$ but do not identify $r$.

# Can Expectations Beat History?

**Overlap** (Krugman, 1991a): initial distributions from which paths lead to *either* asymmetric steady state. For this range, expectations, not history, select the destination.




(a) $q = 0.15$ (b) $q = 0.03$

Same economy and steady states; only the *timing* of amenity spillovers differs ($q = \beta_2$, $\beta_1 + \beta_2$ fixed). (a) $q = .15$: history decides. (b) $q = .03$: paths spiral; the shaded band is the overlap.

inherited origins $\psi$ ▪ historical spillovers $q$ ▪ descendants $\rho$ ▪ notice $H$

# Spirals and Overlap

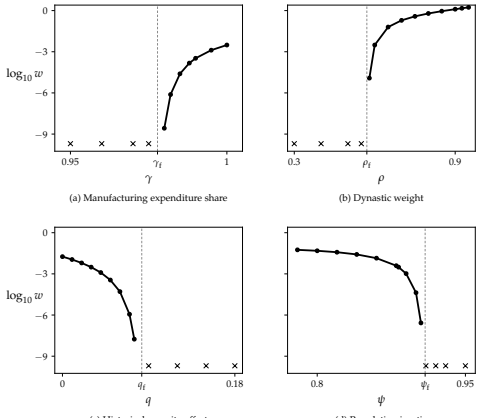
With $q \geq 0$ and $\chi > \chi^*$, paths spiral near symmetry exactly when:

$$0 \leq \left(\sqrt{\psi + mq} - \sqrt{\rho\psi}\right)^2 \; < \; m\,(G^+ - \Theta^+).$$

- Right: $m$ times the **net agglomeration force**.
- Left: the squared gap between the square roots of
  - the weight of inherited settlement $(\psi + mq)$ and
  - the weight of descendants' opportunities $(\rho\psi)$.
- ($q \geq 0$ rules out local indeterminacy, but not Krugman's overlap.)

When the gap is small relative to the net agglomeration force, paths rotate near symmetry. In this example, different expectations can then lead the same initial population toward either center.

# What Widens the Overlap [if time]




(a) Manufacturing expenditure share

(b) Dynastic weight

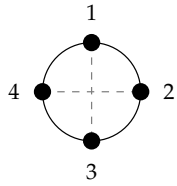
(c) Historical amenity effect

(d) Population inertia

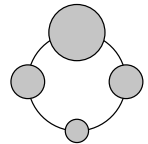
- In these examples, overlap **widens with patience** and **narrows with historical spillovers and inertia**.
- **Immobile demand can remove it**: here, a small fall in $\gamma$ below one stops the rotation, and initial settlement again decides the core.
- Expectations can offset at most $\rho \log \mu$ of the current utility difference, confining overlap near symmetry in this example. Compare the selection barrier in Oyama (2009).

All panels start from $(\varepsilon, \zeta, \gamma, \mu, \theta, \rho, b, q, \chi) = (1, 1, 1, 2, 4, .6, .15, .03, .95)$; dashed guides are local focus–node boundaries.

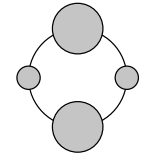
# Four Regions on a Ring: One Center or Two?




(a) Ring geography



One-center $(1, 0, -1, 0)^\top$



Two-center $(1, -1, 1, -1)^\top$

(b) Nonuniform spatial patterns

- Trade and migration freeness decay with ring distance; uniform population is a steady state.
- Two ways to depart from uniformity:
  - a **one-center** pattern ($k = 1$; a shift from region 3 toward region 1, in any orientation),
  - a **two-center** pattern ($k = 2$; regions 1 and 3 against their neighbors).

# Four-Region Eigenvalues

On each pattern, trade and migration act as scalars. The pattern's eigenvalues are:

$$(\chi_1, \chi_2) = (\chi, \chi^2) \qquad \text{and} \qquad (\psi_1, \psi_2) = (\psi, \psi^2).$$

Averaging over neighbors cancels more of the alternating pattern: both trade and migration preserve more of the coarse one.

- Pattern $k$ obeys the two-region equations with $(\chi_k, \psi_k)$, coefficients $p_k, q_k, m_k$, stable root $\lambda_{s,k}$, anticipation factor $r_k$, and

$$G_k^+ = (\zeta + \alpha_1 + \alpha_2)\,\mathcal{M}(\chi_k) - 1 + \beta_1 + \beta_2, \qquad \Theta_k^+ = \frac{1 - \rho\psi_k}{\theta(1 + \psi_k)}.$$

- $S_k \equiv G_k^+ - \Theta_k^+$: the **net agglomeration force**. With $q_k \geq 0$ and $S_k < 0$, the steady-state population multiplier is $\kappa_k = 1/(\Theta_k^+ - G_k^+)$.

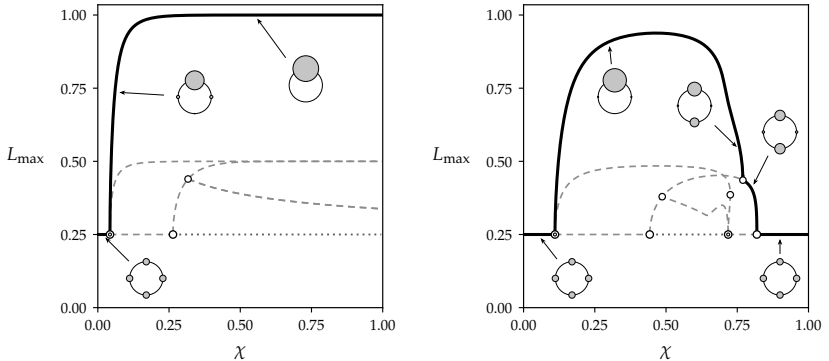
# Which Pattern Does Concentration Favor?

**Proposition** ($\zeta + \alpha_1 + \alpha_2 > 0$, $q_k \geq 0$).
(1) All mobile (Type L): $S_1 > S_2$. One center has the larger multiplier on $S_k < 0$.
(2) Immobile demand (Type LG): $S_2 > S_1$ if and only if

$$G_2^+ - G_1^+ \;>\; \Theta_2^+ - \Theta_1^+ \quad \left( = \frac{(1+\rho)(\psi - \psi^2)}{\theta(1+\psi)(1+\psi^2)} > 0 \right).$$

- Raise the common current amenity elasticity from $S_k < 0$: the larger $S_k$ reaches zero first.
- Migration gives one center the lower threshold. A single-peaked $\mathcal{M}$ can give two centers enough of a goods-market advantage to offset the gap.
- Type L, general $N$: with decreasing spectra, the lowest frequency reaches onset first.

# Endogenous Spatial Configurations as Trade Costs Fall



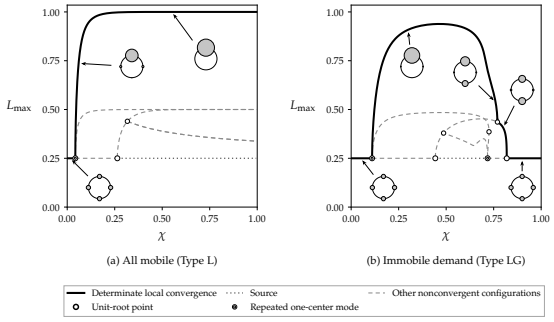

(a) All mobile (Type L)

(b) Immobile demand (Type LG)

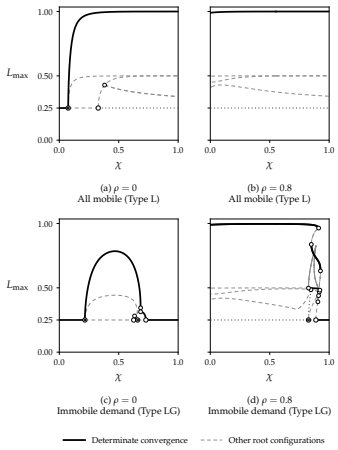
$\rho = .8$, $(\varepsilon, \zeta, \theta, \psi) = (5, .8, 20, .5)$, $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (.2, .1, -.4, .3)$; $\gamma = 1$ in (a), .3 in (b). Illustrative parameters; branches computed over the displayed range.

# Endogenous Spatial Configurations as Trade Costs Fall

- **Type L**: the computed agglomerated branch has one peak. Integration spreads population until the branch meets uniformity.
- **Type LG**: as costs fall, the computed convergent configurations are uniform, two equal centers, two unequal centers, one peak, then uniform again.
- Dispersion forces change which configurations the model can support in the long run, a recurring theme in AMOT.



# Patience Favors the Coarse Pattern [if time]




- $S_k$ rises in $\rho$ with slope $\psi_k/[\theta(1+\psi_k)]$, larger for the pattern migration preserves more: on the ring, one center.
- Greater patience can switch the first critical pattern only toward larger migration eigenvalues.
- As trade costs vanish, the utility effect of concentration tends to .08 for both patterns: below both thresholds under myopia, above both at $\rho = .8$. At low costs, symmetry is stable in the left column and a source in the right.

Rows: Type L and Type LG. All primitives except $\rho$ fixed: $(\varepsilon, \zeta, \theta, \psi) = (5, .8, 5, .75)$, $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (.2, .1, -.32, .3)$.

# An Announced Improvement

**Experiment.** A permanent amenity profile $\boldsymbol{\omega}$ is implemented at $T$ and first learned at $T - H$ ($H$ generations of notice). Only $H$ varies.

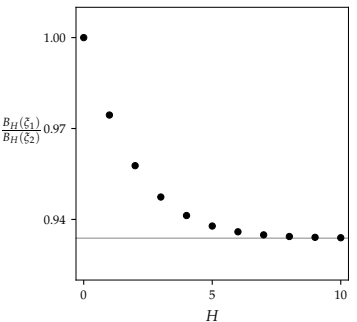
With both patterns stable, the log-population response at $T$ relative to no improvement is

$$\Delta \boldsymbol{\ell}_T = \sum_{k=1}^{2} K_k \, B_H(\xi_k) \, \boldsymbol{\Pi}_k \boldsymbol{\omega}, \qquad K_k = \kappa_k(1 - \lambda_{s,k}), \quad \xi_k = \lambda_{s,k} \, r_k, \quad B_H(x) = \sum_{j=0}^{H} x^j.$$

- Three factors per pattern: the **impact** of a surprise $K_k$, the **accumulation** over the notice period $B_H(\xi_k)$, and the **weight** $\boldsymbol{\Pi}_k \boldsymbol{\omega}$ of the profile on the pattern.
- Each extra period of notice adds another power of $\xi_k$ = anticipation $r_k$ × persistence $\lambda_{s,k}$ of the movement induced earlier. At $H = 0$, $B_0 = 1$: surprise implementation.

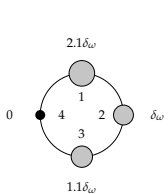
# Notice Reweights Toward the Pattern with the Larger $\xi_k$

- One-center to two-center ratio at $T$, relative to surprise: $B_H(\xi_1)/B_H(\xi_2)$, rising in $H$ iff $\xi_1 > \xi_2$.
- **Type L**: $\xi_1 > \xi_2$ when $\zeta + \alpha_1 > 0$, $\alpha_2 \geq 0$, and $q_2 \geq 0$.
- **Type LG**: $\xi_2 > \xi_1$ is possible (figure: high trade costs).
- Notice leaves $G_k^+$ and $\Theta_k^+$ unchanged and *reweights* the response toward the pattern with larger $\xi_k$.

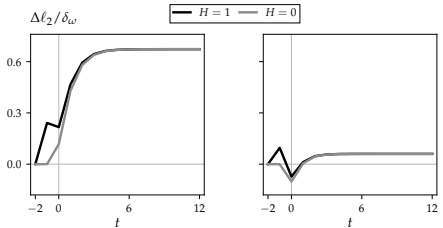



$(\varepsilon, \zeta, \gamma, \chi, \psi, \theta, \rho) = (5, .8, .4, .9, .7, 3, .8)$, $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (.5, .02, -.25, .01)$; dots: $H = 0, \ldots, 10$; gray line: $H \to \infty$.

# Same Improvement with Opposite Regional Effects




(a) Amenity profile



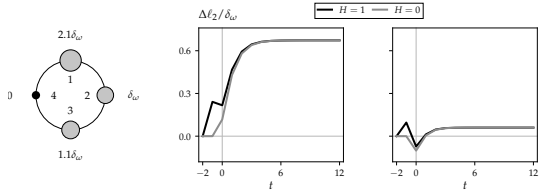

(b) $\gamma = 1$ (Type L)

(c) $\gamma = .4$ (Type LG)

- $\boldsymbol{\omega} = \delta_\omega(2.1, 1, 1.1, 0)^\top$: region 2 does gain directly but its neighbors gain more.
- The one-center component $\propto (1, 1, -1, -1)$ favors region 2; the two-center component $\propto (1, -1, 1, -1)$, with the larger weight, draws population away.
- Which wins depends on how strongly the economy amplifies each pattern.

$(\varepsilon, \zeta, \chi, \psi, \theta, \rho) = (5, .8, .8, .3, 3, .8)$, $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (.1, .02, -.15, .01)$; agricultural labor per region 0 and 1.5.

# A Winner in Type L and a Loser in Type LG




(a) Amenity profile (b) $\gamma = 1$ (Type L) (c) $\gamma = .4$ (Type LG)

- **Type L**: one center amplified more; region 2 gains at implementation and in the long run.
- **Type LG**: the larger two-center loading dominates the nearly equal impacts. Region 2 loses at implementation (with notice: a small gain before $T$, then a loss).
- The reversal already occurs under surprise, and $S_1 > S_2$ in both economies. Regional effects depend on the loadings and the dynamics of both patterns.

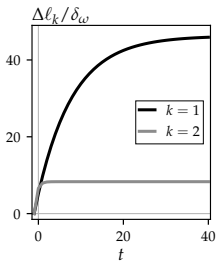
# What Does a Fitted Surprise Path Identify?

A permanent surprise at $T$ moves component $k$ along

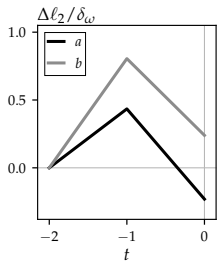
$$\Delta \ell_{k,T+h} = \kappa_k \big(1 - \lambda_{s,k}^{h+1}\big)\, \omega_k, \qquad h \geq 0.$$

- A two-parameter curve: **where it ends** $(\kappa_k)$ and **how fast** $(\lambda_{s,k})$.
  $\Rightarrow$ With known $\omega_k$, it identifies these two objects but leaves $r_k$ undetermined.
- I construct a **matched family** with common geography, goods-market shares, and migration technology. Patience and the four spillover elasticities move together so that $(\kappa_k, \lambda_{s,k})$ stay fixed for both patterns.
- Every member of this family produces the same local population path after *every* permanent surprise, while the anticipation factors vary along the family.

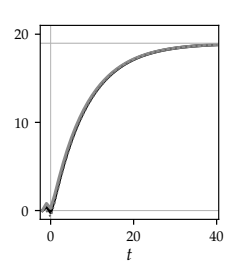
# Same Surprise Paths with Opposite Responses




(a) Permanent surprise




(b) Region 2 (implementation)




(c) Region 2 (common limit)

- (a) Surprise paths of both patterns are identical. (b) One period of notice: region 2 loses in economy $a$ ($\rho = .3$) and gains in $b$ ($\rho = .85$). (c) The long run is common.
- Identical surprise paths can conceal *opposite regional responses* with advance notice.

Type L, $(\varepsilon, \zeta, \gamma, \chi, \psi, \theta) = (3, .3, 1, .4, .43, 3)$; profile $\delta_\omega(4.2, 1, 3.2, 0)^\top$; responses per unit amplitude.

# Evidence That Closes the Prediction Gap

- **News ratio.** With $z_{k,t} = \Delta\ell_{k,t} - \lambda_{s,k}\Delta\ell_{k,t-1}$ and an improvement announced by $T-1$: $z_{k,T-1}/z_{k,T} = r_k$. With $(\kappa_k, \lambda_{s,k})$, it fixes the response at every $H$.
- **A change of known duration.** A one-period surprise known to expire has impact $K_k(1-r_k)$; the shortfall measures $r_k$.
- **More distinct spatial patterns**: with three or more informative patterns, common spillover elasticities impose additional restrictions. **Sign restrictions** on primitives can also narrow the family.
- Validate the *causal prediction* without validating the whole model (Adão *et al.*, 2025); with other shocks in the window, newly induced movements must first be isolated.

# Prediction Bounds for a Region

Over the admissible members of the matched family, compute the smallest and largest implementation responses of the region.

For a connected admissible interval, every intermediate response is attainable.

Previous example, $H = 1$, region 2 per unit amplitude:

| admissible family | patience | region 2 at implementation |
|---|---|---|
| surprise paths only | $\rho \in [.3, .85]$ | $[-.23,\ .24]$: sign open |
| + news ratio | $\rho \in [.7, .8]$ | $[.14,\ .21]$: a gain |
| + $\beta_2 \geq 0$ instead | $\rho \in [.30, .35]$ | $[-.23,\ -.17]$: a loss |

- A news-ratio interval can rule out economies that fit surprise paths yet predict opposite announcement effects.
- What must be observed depends on which regional counterfactual is being reported.

# Three Ingredients of Spatial Dynamics

| | |
|---|---|
| **Dispersion force** | *Which* patterns are preferred when agglomeration occurs:<br>• always one center first in Type L<br>• two separated centers possible in Type LG |
| **Intergenerational linkages** | • *How strongly* each pattern is amplified ($\kappa_k$),<br>• *how long* it persists ($\lambda_{s,k}$), and<br>• *how far ahead* households respond ($r_k$) |
| **History vs expectations** | • *History* ($\psi + mq$): inertia<br>• *Expectations* ($\rho\psi$): acceleration<br>• Together: overlap, timing versus patience, $\xi_k = \lambda_{s,k} r_k$, and what surprise paths leave free |

# Takeaways

1. **Patience** lowers thresholds most for the patterns migration preserves: in particular, one center at the onset of agglomeration. **Notice** moves no threshold and reweights toward the accumulating pattern: e.g., two centers under immobile demand.
2. The same announced improvement can make a region a **winner** under one dispersion mechanism and a **loser** under the other. This echoes the mechanism dependence of spatial predictions in $\mathrm{AMOT}$.
3. Fitting population paths after permanent surprises does not validate the response to an announced change. News ratios and changes of known duration can identify anticipation. Spatial or sign restrictions can narrow the prediction range.

# Open Questions [if time]

- **Heterogeneous steady states.** The symmetric ring is convenient but robustness is an issue. The sign disagreement survives small perturbations of a regular matched economy but not necessarily large ones. Next: response operators at the actual steady state of the Japanese regional system, from observed trade and migration flows.
- **Global selection** beyond the symmetric ring: no potential function here; migration costs confine the overlap near symmetry.
- **Transport-cost shocks**: Transport investments are a natural next application. Their utility effects would enter the linear system through their spatial pattern loadings.
- **Welfare and policy evaluation**: Not attempted here.

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