History, Expectations, and the Geography of Adjustment

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Where do people move when a region is about to change? I study an overlapping-generations spatial economy in which trade, migration frictions, historical spillovers, and concern for descendants link location choices across generations. The geography of adjustment depends on both the spatial reach of dispersion forces and the intergenerational linkages. Positive historical spillovers rule out self-fulfilling expectations near a steady state, yet the same initial state can still lead to different steady states when households weigh descendants' opportunities heavily. Valuing the future and learning about it earlier have different spatial effects: patience favors monocentric spatial patterns that migration frictions keep in place across generations, whereas earlier notice can give greater weight to patterns with several separate centers. Finally, a model that fits adjustment to unanticipated changes need not predict adjustment to announced ones, even in sign. News and changes of known duration can supply the missing information.

Keywords: spatial dynamics; expectations; intergenerational spillovers; migration; spatial scale.

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1 Introduction

Theories of agglomeration explain the concentration of economic activity through the interaction of agglomeration and dispersion forces. These forces also constrain the geographical predictions a model can produce. A dispersion force that spreads residents around an existing economic center has different implications from one that keeps several centers apart. Theories have shown that this distinction matters for the number, spacing, and extent of spatial agglomerations and for the qualitative direction of spatial counterfactuals. Fitting an observed population distribution does not by itself establish that a model contains the endogenous forces relevant to a proposed counterfactual shock.

In a dynamic economy, regional advantages also pass between generations. Migration frictions preserve part of the inherited population distribution. Past settlement affects current regional productivity and amenities. Households may choose locations partly for the opportunities available to descendants. How do these intergenerational linkages interact with the spatial reach of dispersion forces to determine where people move? What must we observe to discipline a model's prediction of that adjustment?

Consider a lasting improvement in regional living conditions that becomes known before it is realized. Households can settle in anticipation, and their destinations become the next generation's origins. I represent this situation by an exogenous amenity profile, with one period equal to one generation. The prediction of interest is the population response of each region, including during the interval between revelation and implementation. The purpose of this theoretical experiment is to understand what kind of information can support such a prediction in a dynamic quantitative spatial model (QSM).

The geography of adjustment depends on both the spatial reach of dispersion forces and the way inherited settlement, historical spillovers, and descendants' opportunities enter current location choice. What matters is how the two interact, and two comparisons make the interaction concrete.

First, increasing the weight households place on descendants and giving them earlier notice of the same improvement have different spatial effects. Greater patience lowers the utility advantage a concentration needs to sustain itself, and lowers it most for population patterns that migration preserves more strongly. Earlier notice leaves every force and threshold unchanged and instead gives each response more time to accumulate

before implementation, an accumulation that depends on both the response to future utility and the survival of population changes already induced. On a four-region ring, greater patience favors the coarse one-center pattern at the onset of endogenous agglomeration, whereas earlier notice can shift the composition of the response toward the finer two-center pattern. The same improvement can then make a region a winner under one dispersion mechanism and a loser under another.

Second, the entire local population path after a permanent surprise improvement can coincide across economies whose responses to advance notice differ. I construct a family with common trade and migration geography in which patience and spillovers vary jointly so as to preserve the long-run population multipliers and recovery rates. These fix every local population response to a permanent surprise but leave the anticipation factors free, and differences across spatial patterns can then reverse the predicted gain or loss of a particular region under the same announced amenity profile. News ratios, changes of known duration, restrictions from richer geography, and sign restrictions on spillovers supply the missing information and can establish the sign of a region's response without identifying every structural parameter.

The model combines production, trade, and market clearing from Kucheryavyy, Lyn and Rodríguez-Clare (2024), henceforth KLR, with the contemporaneous and historical spillovers and dynastic location choice of Allen and Donaldson (2020, 2022), henceforth AD. An all-mobile economy and an economy with immobile agricultural demand carry different dispersion forces, corresponding to the local- and global-dispersion cases in AMOT; I call them Type L and Type LG. Section 2 sets out the economy. Section 3 develops the two-region case: goods-market forces interact with settlement externalities, and comparing the utility advantage that concentration generates with the advantage needed to sustain it decides when symmetry gives way to endogenous agglomeration. Section 4 isolates forward-looking behavior. Concern for descendants makes concentration easier to sustain, anticipation and recovery are governed by two temporal roots, and shifting amenity spillovers from current to inherited population leaves every steady state unchanged while slowing recovery and weakening anticipation. The same shift can change which destinations an initial population distribution can reach: paths toward either of two asymmetric steady states exist when the gap between the weight of inherited settlement and the weight of descendants' opportunities is small relative to the net agglomeration force.

Section 5 turns to four regions, which distinguish a single center from two separated

centers. The Type L economy favors the one-center pattern both in amplifying a given improvement and at the onset of endogenous agglomeration; immobile demand can reverse this ordering by keeping production near separated consumers. Numerical solutions of the nonlinear equations show the configurations each mechanism supports: convergent separated centers with immobile demand and single-peaked agglomerations in the all-mobile economy. A fixed-primitives comparison shows that greater patience can remove convergence around uniform population even at low trade costs. Sections 6 and 7 then develop the two comparisons above: the spatial composition and regional incidence of an announced improvement, and what evidence validates a model's prediction of it. The symmetric ring makes the equivalence of surprise paths exact, and a perturbation result shows that the regional disagreement survives small heterogeneity. Section 8 concludes.

Related literature

Theories following Krugman (1991b) explain agglomeration through the balance between increasing returns, market access, and various dispersion mechanisms (Fujita et al., 1999; Fujita and Thisse, 2013). AMOT organize this balance by the spatial reach of dispersion forces and show how the type of dispersion force in a quantitative spatial model shapes its counterfactual predictions. I extend this approach to a dynamic economy with intergenerational linkages. I use the spectral methods developed by Akamatsu, Takayama and Ikeda (2012) to compare the geographical patterns affected by the same economic forces.

The role of history and expectations in selecting an outcome is a longstanding question. Krugman (1991a) studies external economies with adjustment costs, and Matsuyama (1991) studies industrialization with irreversible choices in an overlapping-generations economy. Fukao and Bénabou (1993); Baldwin (2001); Ottaviano (2001) analyze related forward-looking migration problems. The "overlap" example uses this established distinction between inherited conditions and expectations; its role is to show what spillover timing changes at fixed steady states. Global selection results such as those of Oyama (2009b,a), following Matsui and Matsuyama (1995), require additional potential and regularity conditions. Takeda and Yamagishi (2026) bring the selection question to data. Their estimated model of Hiroshima finds that contemporaneous density effects dominate lagged effects and also admits a computed equilibrium path

from the observed post-bombing distribution in which the center does not recover. The comparison illustrates how expectations can matter for the observed recovery.

Overlapping-generations spatial models also distinguish benefits received at different dates. Takahashi (2022) studies workers and retirees with different location incentives, while Fujishima and Oyama (2023) connect time preference, spillover networks, and global stability. Suzuki (2026) quantifies how age-specific mobility and forward-looking local capital shape regional responses to unanticipated local shocks in an aging economy. I use the dynastic structure in AD, which distinguishes inherited origins, historical utility effects, and descendants' opportunities, to vary spillover timing while holding steady-state incentives fixed.

As surveyed by Desmet and Parro (2025), dynamic quantitative spatial models provide a unified framework for migration and trade (Desmet et al., 2018; Caliendo et al., 2019; Peters, 2022). Kleinman, Liu and Redding (2023) develop a spectral representation of transitions with forward-looking migration and capital accumulation, including anticipated fundamentals. The two-region dynamics in Section 4 are a special case of their linearized system, used here to express the recovery rate and the anticipation factor in terms of spillover timing and patience. Fan, Hong and Parro (2025) study evolving, uncertain, and heterogeneous beliefs and the information in belief data. I assume perfect foresight from the revelation date onward to isolate how dispersion forces and intergenerational linkages constrain the spatial effect of notice.

In the broader literature on identification, Magnac and Thesmar (2002) characterize identification in dynamic decision processes, and Kalouptsidi, Scott and Souza-Rodrigues (2021) distinguish identifying a counterfactual from identifying the underlying structure. Adão, Costinot and Donaldson (2025) make the same distinction the basis of a test: exogenous shifters of an observed change can test a quantitative model's causal prediction for the change, in the presence of other shocks, without testing the model as a whole. In Section 7, I examine what that distinction means for regional predictions when the observed and the counterfactual changes differ in anticipation.

2 The spatial economy

The model combines the within-period goods markets of KLR with the spillovers and dynastic location choice of AD. It keeps separate the inherited origins of descendants, the effects of past settlement on current productivity and amenities, and the weight

households place on descendants' opportunities.

There are $N \geq 2$ regions and periods $t = 0, 1, \ldots$, each interpreted as a generation. Each generation contains a unit mass of mobile manufacturing workers. Their distribution is $\boldsymbol{L}_t = (L_{it})_{i=1}^N$, with $\boldsymbol{L}_t \in \triangle := \{\boldsymbol{L} > \boldsymbol{0} : \sum_i L_i = 1\}$. Region i also contains a fixed mass $\overline{L}_i^{\mathrm{A}} > 0$ of immobile agricultural workers, when the agricultural sector is present.

Each mobile worker has one descendant, who inherits the parent's location as an origin. In period t, $\boldsymbol{L}_{t-1}$ is predetermined. Migration determines $\boldsymbol{L}_t$, after which production and trade clear and period utility is realized. Location choice incorporates the opportunities available to the next generation. Aggregate paths are perfectly foreseen conditional on the information available when choices are made, and idiosyncratic location tastes average out.

2.1 Production, trade, and intergenerational spillovers

Building on KLR and AD, the within-period environment combines goods markets with contemporaneous and historical spillovers. Region i produces an Armington manufacturing variety at unit cost $c_{it} = w_{it}^{\zeta} P_{it}^{1-\zeta} / a_{it}$, where w_{it} is the manufacturing wage, P_{it} is the manufacturing price index, and a_{it} is productivity. Here, following Krugman and Venables (1995), manufacturing varieties also serve as intermediate inputs. The labor cost share is $\zeta \in (0, 1]$, and the remaining share is spent on intermediates. Shipping from origin i to destination j requires $\tau_{ji} \geq 1$ units per unit delivered. Manufacturing varieties have substitution elasticity $\sigma = 1 + \varepsilon > 1$, so that the trade elasticity is $\varepsilon > 0$.

Final spending assigns share $\gamma \in (0, 1]$ to manufactures and the remaining share to freely traded agriculture. A mobile worker's indirect utility in period t is

$$U_{it} = b_{it} \frac{w_{it}}{P_{it}^{\gamma} (w_t^{\mathrm{A}})^{1-\gamma}}, \tag{1}$$

where w_t^{A} is the agricultural wage common across regions. When $\gamma < 1$, I use agriculture as numeraire and set $w_t^{\mathrm{A}} = 1$. Because agricultural workers are immobile, they anchor part of manufacturing demand to fixed locations. Manufacturing intermediates, in turn, connect local production costs to demand for manufactures. When $\gamma = 1$, agriculture is absent and I normalize aggregate manufacturing income to unity.

Following AD, I assume externalities of the form

$$a_{it} = \overline{a}_i L_{it}^{\alpha_1} L_{i,t-1}^{\alpha_2} \qquad \text{and} \qquad b_{it} = \overline{b}_i L_{it}^{\beta_1} L_{i,t-1}^{\beta_2} e^{\omega_{it}}, \tag{2}$$

where $\bar{a}_i, \bar{b}_i > 0$ are fixed regional fundamentals. Here, ω_{it} is an exogenous log-amenity shift, zero unless an amenity change is specified. Section 6 considers an announced amenity improvement described in the introduction. Productivity affects wages and prices through trade, while amenities enter utility directly. The elasticities α_1, β_1 govern current effects, and α_2, β_2 govern historical (intergenerational) spillovers. Inherited knowledge and neighborhood quality can make historical effects positive, whereas delayed congestion or displacement can make them negative. Even without historical spillovers, inherited origins link successive population distributions.

Appendix A lists the KLR market clearing conditions and derives the utility response to population redistribution. At a regular interior equilibrium for which $L_{it} > 0$ for all i and t, those conditions determine utility from current and inherited population.

2.2 Forward-looking location choice

Let $\mu_{ij} \geq 1$ be the finite migration cost from inherited origin i to destination j, with $\mu_{ii} = 1$. A worker draws independent multiplicative Fréchet location tastes with shape $\theta > 0$ and chooses the destination, maximizing taste-adjusted location value. Migration probabilities and population conservation imply

$$L_{jt} = \sum_i \pi_{ij,t} L_{i,t-1} \qquad \text{where} \qquad \pi_{ij,t} = \frac{\mu_{ij}^{-\theta} V_{jt}^{\theta}}{\sum_h \mu_{ih}^{-\theta} V_{ht}^{\theta}}. \tag{3}$$

Larger θ means less dispersed tastes and a sharper response to differences in systematic value $\{V_{it}\}$. Migration costs make inherited origins matter even at equal values. Each origin allocates its workers according to destination values after migration costs; the next generation inherits the resulting distribution.

With dynastic weight $\rho \in [0, 1)$, location values satisfy

$$V_{it} = U_{it} \left(\sum_j \mu_{ij}^{-\theta} V_{j,t+1}^{\theta} \right)^{\rho/\theta}. \tag{4}$$

The continuation term is the next generation's opportunity value from origin i. At $\rho = 0$, location choice is myopic and the current generation does not care their decendants. At $\rho > 0$, a location becomes attractive partly through its access to future generations' opportunities. The same location can therefore be chosen for its current utility and for the options it offers a descendant who starts there.

An interior *dynamic equilibrium* consists of positive population and value paths satisfying market clearing, Equations (1), (3) and (4), from a given initial population distribution and under the specified information path. For $\rho > 0$, I impose $\lim_{J\to\infty} \rho^{J+1} \max_i |\log V_{i,J+1}| = 0$ to exclude value bubbles. A *steady state* is a dynamic equilibrium that has constant population and values.

3 Endogenous forces and dynamics

This section explores the endogenous workings of the model by studying the dynamics around an interior steady state in a symmetric two-region economy. Consider two regions with identical fundamentals, where uniform population, $L_{1t} = L_{2t} = 1/2$, is a steady state. A small redistribution toward one region changes its goods-market equilibrium as well as amenities today and, through inherited population, those of the next generation. We follow such a redistribution to see whether the advantage fades or reinforces itself around the symmetric equilibrium, and also examine asymmetric steady states when endogenous agglomeration occurs.

3.1 Population changes and spatial interactions

Define relative population as the log ratio $\ell_t = \log(L_{1t}/L_{2t})$. Region 1 is larger when $\ell_t > 0$. Since total population is fixed, all local population changes follow the direction $\boldsymbol{e} = (1, -1)^\top$, and ℓ_t measures the resulting imbalance.

Spatial interaction attenuates the population advantage ℓ_t. To see this, suppose access to population takes the form $A_i = L_i + \phi L_j$ for $j \neq i$, with $\phi \in (0, 1)$ bein a cross-origin access parameter. A small shift toward region 1 $\ell_t > 0$ raises A_1 and lowers A_2, but this is not one to one because each region also accesses the other's population. Consequently, the log accessibility difference, $\log(A_1/A_2)$, changes by only $(1-\phi)/(1+\phi) \in (0, 1)$ times the relative population ℓ_t.

This is represented by the eigenvalue of the *proximity matrix* (Matsuyama, 2017):

$$\overline{\boldsymbol{T}}(\phi)\boldsymbol{e} = \frac{1-\phi}{1+\phi}\boldsymbol{e} \qquad \text{where} \qquad \overline{\boldsymbol{T}}(\phi) = \frac{1}{1+\phi}\begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix}. \tag{5}$$

The row-normalized proximity matrix preserves less of the difference when cross-region access is easier. The multiplier $(1-\phi)/(1+\phi)$ approaches one as cross-region access

vanishes and is zero under frictionless access. This eigenvalue representation underlies the spectral approach for spatial models (Akamatsu et al., 2012; Kleinman et al., 2023). For trade and migration in the two-region economy, applying this relationship to their respective proximity matrices $[\tau_{ij}^{-\epsilon}]$ and $[\mu_{ij}^{-\theta}]$ yields two basic multipliers,

$$\chi = \frac{1-\tau^{-\varepsilon}}{1+\tau^{-\varepsilon}} \qquad \text{and} \qquad \psi = \frac{1-\mu^{-\theta}}{1+\mu^{-\theta}}, \tag{6}$$

both of which rise with their respective symmetric bilateral cost $\tau = \tau_{12} = \tau_{21}$ and $\mu = \mu_{12} = \mu_{21}$. The trade eigenvalue $\chi \in (0,1)$ measures basic attenuation of population advantage ℓ_t in goods market access as discussed above. On the other hand, as discussed below, the migration eigenvalue $\psi \in (0,1)$ measures population inertia due to migration frictions: at equal location values, the fraction ψ of an inherited population difference ℓ_{t-1} survives into the next generation.

3.2 Endogenous location incentives

We now examine the endogenous forces in our model by considering marginally departure from the symmetric steady state, so that $\ell_t \neq 0$ and $\ell_{t-1} \neq 0$. Define the corresponding log differences in period utility, location value, and exogenous amenities as follows: $u_t = \log(U_{1t}/U_{2t})$, $v_t = \log(V_{1t}/V_{2t})$, and $\omega_t = \omega_{1t} - \omega_{2t}$. Linearized at the symmetric steady state, the within-period equilibrium conditions imply

$$u_t = p\ell_t + q\ell_{t-1} + \omega_t, \tag{7}$$

with contemporaneous and historical utility effects

$$p = (\zeta + \alpha_1)\mathcal{M}(\chi) - 1 + \beta_1, \tag{8}$$

$$q = \alpha_2\mathcal{M}(\chi) + \beta_2. \tag{9}$$

Here, $\mathcal{M}(\chi) > 0$, derived in Appendix A, is the real-income response coefficient representing the KLR goods-market structure. It summarizes how wages and prices adjust to labor supply and productivity, and depends on χ and the goods-market parameters $(\varepsilon, \zeta, \gamma)$. To interpret p, shift workers toward region 1 while holding inherited population fixed. Labor supply and current productivity change regional income and prices through $(\zeta + \alpha_1)\mathcal{M}(\chi)$; the term -1 converts regional income to income per worker,

and β_1 adds the amenity effect. When $p > 0$, concentration raises relative utility. To interpret q, instead increase inherited population at a fixed current distribution. Its productivity effect operates through goods markets, while its amenity effect enters utility directly.

Linearizing migration and the value recursion gives

$$\ell_t = \psi \ell_{t-1} + m v_t, \qquad m = \theta(1 - \psi^2) > 0, \tag{10}$$

$$v_t = u_t + \rho \psi v_{t+1}. \tag{11}$$

The first equation distinguishes the inherited difference preserved by migration frictions (the first term) and the additional population attracted by a value advantage (the second term). For instance, when location values are equal ($v_t = 0$), we have $\ell_t = \psi \ell_{t-1}$, so ψ represents basic population inertia under Fréchet migration dynamics that is present without forward-looking choice. The coefficient m is the responsiveness to a value difference. Both coefficients come from the same migration technology: at fixed θ, higher migration costs raise ψ and lower m. The second equation captures descendants' opportunities. A future advantage in region 1 benefits descendants from both origins, but more from origin 1; ψ is the size of this difference, and ρ weights it. With frictionless migration, $\psi = 0$, both origins offer equal access to future locations, while historical spillovers can still matter through q.

3.3 When does a population advantage sustain itself?

A population advantage persists only if the utility gain it generates suffices to maintain it under forward-looking location choice. To make this balance visible, we eliminate location values and obtain a single equation in relative population across three generations. The migration equation gives $v_t = (\ell_t - \psi \ell_{t-1})/m$. Substituting this expression at t and $t+1$ into the value recursion, and using the utility relation, gives, for $0 < \rho, \psi < 1$,

$$\rho \psi \ell_{t+1} - (1 - mp + \rho \psi^2)\ell_t + (\psi + mq)\ell_{t-1} = -m\omega_t. \tag{12}$$

This second-order difference equation relates settlement in three generations. Current and historical utility effects enter through p and q, while forward-looking choice introduces the next generation's population through $\rho\psi$. Given the initial population distribution, today's value must be consistent with the future amenity path. In the absence

of an exogenous difference, a path proportional to λ^t has growth factor satisfying

$$Q(\lambda) = \rho\psi\lambda^2 - (1 - mp + \rho\psi^2)\lambda + \psi + mq = 0. \tag{13}$$

A root at $\lambda = 1$ describes a population difference that persists unchanged, $\ell_t = \ell_{t-1} = \ell$. Along this path, the generated utility advantage is $u = G^+\ell$, where

$$G^+ = p + q = (\zeta + \alpha_1 + \alpha_2)\mathcal{M}(\chi) - 1 + \beta_1 + \beta_2. \tag{14}$$

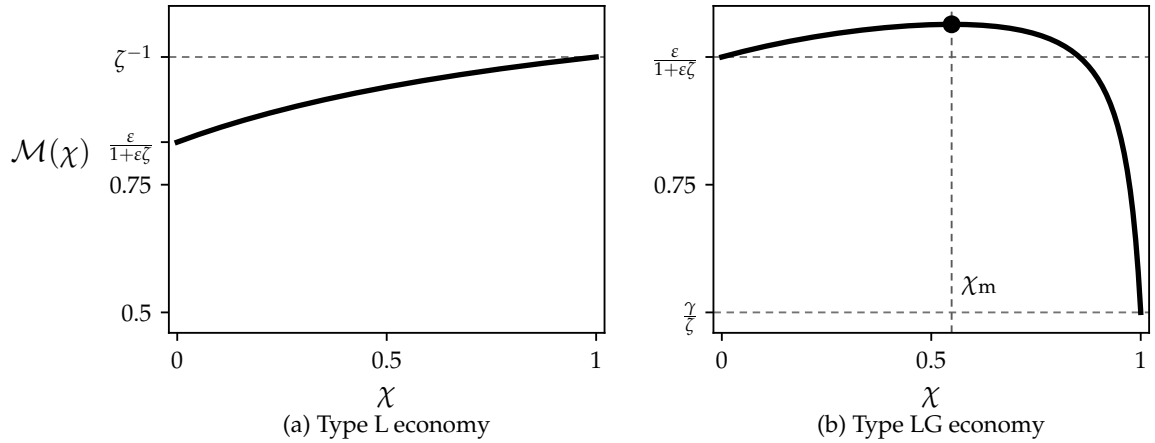
To sustain the same population difference, migration requires $v = (1 - \psi)\ell/m$, and the value recursion requires $u = (1 - \rho\psi)v$. The utility effect needed to sustain the difference is thus $u = \Theta^+\ell$, where

$$\Theta^+ = \frac{1 - \rho\psi}{\theta(1 + \psi)} > 0. \tag{15}$$

Thus, a persistent agglomeration must balance two forces so that $G^+ = \Theta^+$, for which case we indeed have $Q(1) = m(G^+ - \Theta^+) = 0$. Below this threshold, the utility gain generated by a lasting concentration falls short of what forward-looking location choice requires to maintain it. With frictionless migration and myopic choice, $\psi = \rho = 0$, so the boundary reduces to $G^+ = 1/\theta$, where greater θ reduces required utility gain. In general, migration frictions lower the required utility effect through $1 + \psi$; when households value descendants, the continuation term $\rho\psi$ lowers it further.

Suppose historical utility effects are nonnegative, $q \geq 0$. When $G^+ < \Theta^+$, $Q(0) > 0 > Q(1)$ gives two roots $0 < \lambda_s < 1 < \lambda_u$: symmetry is a *positive saddle*. One direction decays toward symmetry and the other grows away from it. Inherited population is predetermined, while location values can adjust to select the convergent path. Each nearby initial distribution therefore admits a unique local equilibrium path converging monotonically to symmetry. I call symmetry *stable* in this local saddle-path sense. Raising p across the boundary at fixed q instead makes symmetry a *source*, with both roots outside the unit circle, until a possible later crossing at -1. I call symmetry *unstable* when it is such a local source: no nonconstant local path converges to it. There is no local equilibrium indeterminacy when $q \geq 0$, since the root product rules out two stable roots: $\lambda_s\lambda_u = (\psi + mq)/(\rho\psi) \geq 1/\rho > 1$.

Here, **AD**'s distinction between equilibrium-path uniqueness and equilibrium persistence appears in two separate objects. The comparison of G^+ with Θ^+ determines



(a) Type L economy (b) Type LG economy

Figure 1: Goods-market responses with mobile and immobile demand.

Note: Panel (a): $(\varepsilon, \zeta, \gamma) = (5, 1, 1)$, all mobile. Panel (b): $(5, .8, .4)$, with immobile demand. Higher χ means higher trade costs.

the local saddle/source boundary, whereas the division of $G^+ = p + q$ between contemporaneous and historical effects changes the temporal roots, and hence the convergence rate, even when that boundary is unchanged.

3.4 Two dispersion mechanisms

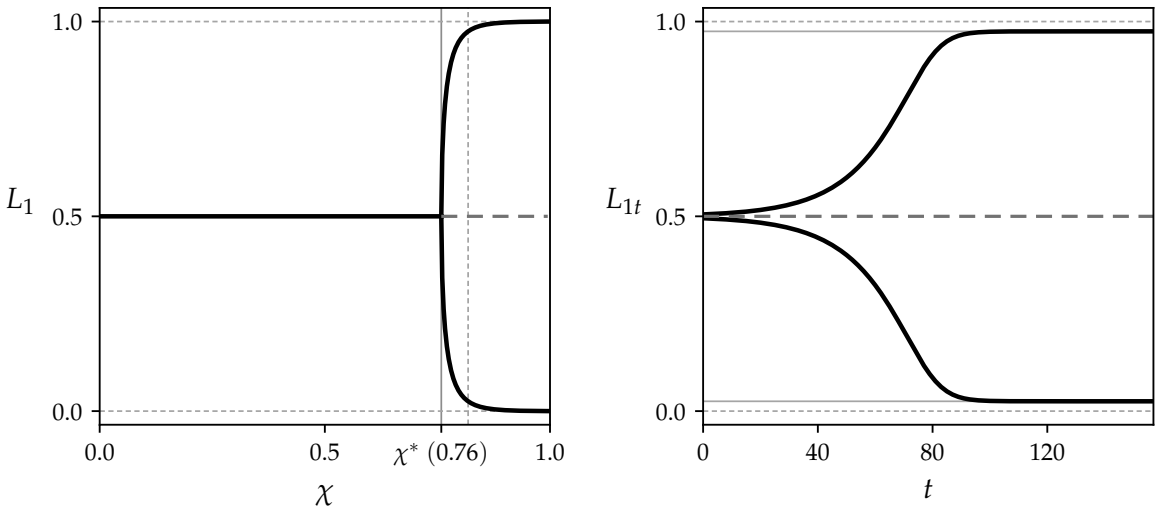
Trade costs move the boundary $G^+ = \Theta^+$ only through the goods-market response coefficient $\mathcal{M}$, so its shape in χ decides where concentration can sustain itself as well as which dispersion mechanism limits it. Set $\alpha^+ = \alpha_1 + \alpha_2$ and $\beta^+ = \beta_1 + \beta_2$, and assume $\zeta + \alpha^+ > 0$ so the steady-state utility effect follows the shape of $\mathcal{M}$.

In the "all-mobile" economy without the agricultural sector ($\gamma = 1$), we have

$$\mathcal{M}(\chi) = \frac{\varepsilon(1+\chi) + \chi}{1 - (1-\zeta)\chi + \varepsilon\zeta(1+\chi)}. \tag{16}$$

This coefficient increases strictly with χ, as illustrated in Fig. 1(a). In this economy, concentrating workers increases the supply of their region's variety and worsens its terms of trade. The real-income loss is stronger at low trade costs, when the region sells more outside its own market. Higher trade costs weaken this dispersion force. Demand moves with workers, and a region losing population has no immobile consumer base pulling manufacturing back toward it.

With agriculture, $\gamma < 1$, demand remains tied to immobile workers. At high trade costs, supplying them from a distant production center is expensive, encouraging manufacturing to remain dispersed. If $\gamma < 2\varepsilon$, this additional dispersion force makes



(a) Steady-state populations

(b) Nonlinear adjustment

Figure 2: Endogenous concentration in two identical regions.

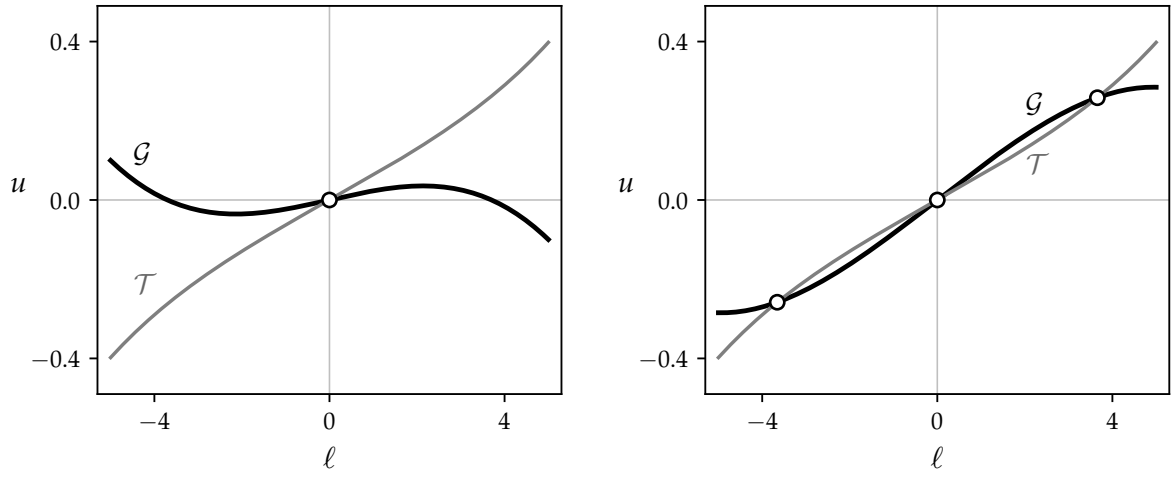
Note: $\varepsilon = \zeta = \gamma = 1$, $\alpha_1 = \alpha_2 = 0$, $(\beta_1, \beta_2) = (.12, .03)$, and $(\mu, \theta, \rho) = (2, 4, .6)$. Panel (a): solid branches are positive saddles; the dashed gray branch is a source. Panel (b): paths at $\chi = 9/11 \approx .82$ begin from different initial population distributions near symmetry. Thin vertical guides mark χ^* and panel (b)'s trade cost; horizontal guides mark symmetry and the asymmetric steady-state populations.

$\mathcal{M}$ single-peaked, as Appendix A.3 proves: concentration generates its largest utility advantage at intermediate trade costs, as Fig. 1(b) illustrates. Intermediate inputs affect the level and peak, but immobile demand supplies the additional mechanism.

In the spirit of AMOT, assuming $\zeta + \alpha^+ > 0$, I call the all-mobile specification *Type L* and the specification with $\gamma < 1$ and $\gamma < 2\varepsilon$ *Type LG*. The Type L economy corresponds to an economy with only local dispersion force, which spreads activity around an existing economic center. The Type LG economy corresponds to one with a global dispersion force that may keep multiple separated centers apart. Section 5 develops their spatial implications.

3.5 Asymmetric steady states

While the loss of convergence around symmetry does not establish that asymmetric steady states exist in general, our economy indeed admits stable asymmetric steady states when the symmetry is unstable. For illustration, consider a Type L all-mobile economy. Let production have no intermediates or productivity spillovers, and let both current and inherited population improve amenities ($\beta_1, \beta_2 > 0$) for simplicity. The amenity gain competes with the terms-of-trade loss from concentration. In Fig. 2, low trade costs make symmetry the unique steady state. Beyond χ^*, symmetry becomes a



(a) $\chi = \frac{2}{3}$ (b) $\chi = \frac{9}{11}$

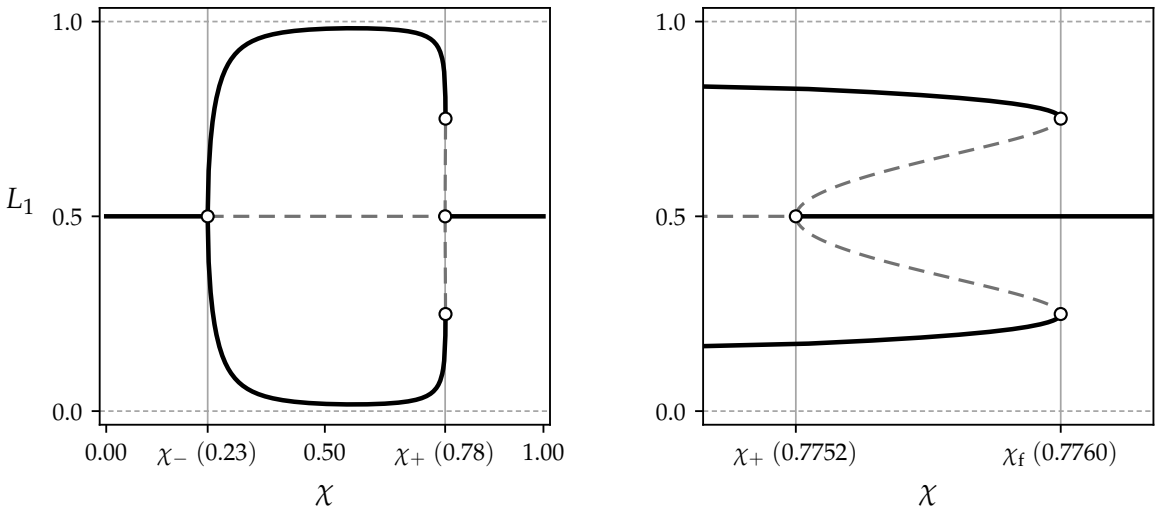
Figure 3: Generated and required utility advantages.

Note: $\ell = \log(L_1/L_2)$ is the relative population held constant across generations, and $u = \log(U_1/U_2)$ is the log utility advantage. Black: generated advantage $\mathcal{G}(\ell)$; gray: required advantage $\mathcal{T}(\ell)$. Open circles: steady states. Parameters other than χ follow Fig. 2.

source and hence unstable. Two mirror-image asymmetric steady states emerge, and they are stable. The illustrated paths in panel (b) start from small initial population imbalances of opposite signs. History matters here, because the initially larger region attracts further and further population toward an asymmetric steady state.

Away from symmetry, stronger concentration increases the real-income loss relative to the amenity benefit. Figure 3 shows how this limits concentration. At a relative population ℓ held constant across generations, let $\mathcal{G}(\ell)$ be the log utility advantage generated by settlement and $\mathcal{T}(\ell)$ the advantage required to sustain that distribution under migration and forward-looking choice. Sustaining unequal populations requires the generated advantage to compensate for the tendency of the migration dynamics to disperse residents. Their intersections are steady states, and their slopes at symmetry satisfy $\mathcal{G}'(0) = G^+$ and $\mathcal{T}'(0) = \Theta^+$, connecting the local analysis at the symmetric state to the incentives that support asymmetric steady states. Appendix B.3 provides a full nonlinear analysis for this economy.

Figure 4 illustrates the additional role of global dispersion in the Type LG economy. Immobile agricultural consumers sustain demand in both regions, making concentration of manufacturing in one region costly when trade costs are high, as in Krugman (1991b). In AMOT's terminology, this force promotes dispersion across regions, whereas a local dispersion force limits concentration within an agglomeration. In this numerical



(a) Steady-state populations

(b) Coexistence range (enlarged)

Figure 4: Type LG steady-state branches in two regions.

Note: $(\varepsilon, \zeta, \gamma) = (2, .6, .2)$, $(\beta_1, \beta_2) = (.4325, .03)$, and $(\mu, \theta, \rho) = (2, 4, .6)$, with unit total mobile population and agricultural labor equal to 2 in each region. Solid branches are positive saddles; dashed gray branches are sources; open circles mark nonhyperbolic points. Panel (b) enlarges the narrow window for the coexistence of five steady states.

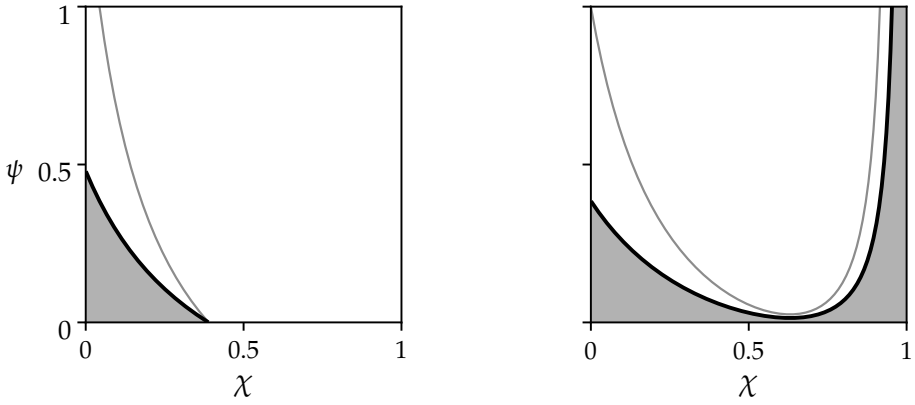
example, symmetry is stable at low and high trade costs, while asymmetric steady states are stable at intermediate costs. A narrow interval also contains five interior steady states. Section 5.3 extends the comparison to four regions, where the number and spacing of centers reveal the different spatial roles of these dispersion forces.

4 The role of forward-looking behavior

Forward-looking location choice enters these dynamics through several distinct margins: the weight households place on descendants, the generation in which settlement affects utility, and the date at which a change becomes known. Each alters present location choice differently. In this section, I ask in turn whether concern for descendants lowers the threshold at which agglomeration kicks in, how a known future change moves people before it arrives, and what distinguishes spillover timing from patience, including the destinations an initial population distribution can reach.

4.1 Stability and the dispersion mechanism

Does concern for descendants make concentration easier to sustain, and does the answer depend on the dispersion mechanism? Holding the within-period environment and



(a) Type L economy ($\gamma = 1$)

(b) Type LG economy ($\gamma = .5$)

Figure 5: Stability of symmetry under myopic and forward-looking location choice.

Note: Dark gray: stable; white: unstable. Solid black: persistent-agglomeration boundary at $\rho = .8$; thin gray: the corresponding myopic boundary at $\rho = 0$. The Type L panel uses $(\varepsilon, \zeta, \gamma) = (5, 1, 1)$ and the Type LG panel $(5, .8, .5)$. Both set $(\theta, \alpha_1, \alpha_2, \beta_1, \beta_2) = (5, .20, .10, -.30, .30)$ whereby $q > 0$.

migration technology fixed, Fig. 5 compares myopic and forward-looking choice. The axes are the trade and migration eigenvalues in Eq. (6), whereas trade costs rise to the right and migration costs upward. In dark gray, symmetry is stable: it is a positive saddle, and each nearby initial distribution admits a unique local equilibrium path converging monotonically to symmetry. In white, symmetry is unstable because it is a local source. The black and gray curves mark the boundary of persistent agglomeration $G^+ = \Theta^+$ discussed in Section 3.3, under forward-looking choice ($\rho = .8$) and myopic choice ($\rho = 0$), respectively.

The two axes act on the boundary $G^+ = \Theta^+$ through different objects. Trade costs enter only the generated utility advantage G^+, through $\mathcal{M}$, whereas migration frictions and patience enter only the required advantage Θ^+. Consider first the horizontal direction. In the all-mobile Type L economy of panel (a), $\mathcal{M}$ increases in χ, so for fixed migration costs symmetry is unstable when trade costs are high. In the Type LG economy with immobile demand in panel (b), the single-peaked $\mathcal{M}$ instead confines instability to intermediate trade costs, so symmetry remains stable at both low and high trade costs.

Consider next the vertical direction. In both panels, instability occurs when migration costs are high, because migration frictions lower the required advantage Θ^+ without affecting G^+: when frictions preserve most of an inherited population difference, only a small location-value advantage is needed to keep the remainder in place. A concentration that would dissolve under easy migration can therefore sustain itself

when migration is costly.

Patience lowers Θ^+ further. Within either environment, greater dynastic weight ρ reduces the convergence region because the utility required to sustain population deviation decreases in ρ:

$$\Theta^+(\rho) = \Theta^+(0) - \frac{\rho\psi}{\theta(1+\psi)} \qquad \text{where} \qquad \Theta^+(0) = \frac{1}{\theta(1+\psi)} > 0. \tag{17}$$

The reduction is proportional to ψ and vanishes under frictionless migration. A concentration that cannot sustain itself under myopic choice can therefore become self-reinforcing under forward-looking choice. The area between the black and gray boundaries consists of such economies: the symmetric steady state loses its convergent path as the dynastic weight rises from 0 to 0.8 in this example.

The mechanism behind the $\rho\psi$ term is descendants' opportunities. Along a persistent population difference, the location-value recursion gives $v = u/(1-\rho\psi)$. Settling in a region with a lasting utility advantage also gives the next generation better access to future opportunities. Households take this continuation benefit into account today, raising the location-value advantage associated with a given period-utility difference. Consequently, a smaller current utility advantage suffices to maintain the same concentration. Expectations can amplify an advantage generated by settlement itself, even without an announced change in fundamentals.

Migration frictions thus lower Θ^+ from two channels: directly, through population inertia, and through their interaction with patience, because costly migration makes descendants' opportunities depend more on their parents' settlement. With $\psi = 0$, both origins offer identical future access, so patience does not change the threshold. A larger ψ also lowers the migration response m; a lower agglomeration threshold therefore need not mean faster movement.

4.2 Anticipation under convergent adjustment

Announced changes are the second margin forward-looking behavior affects the economy. We restrict attention to the positive-saddle region $G^+ < \Theta^+$, where $0 < \lambda_s < 1 < \lambda_u$. Around a stable symmetric steady state, the answer takes the standard form of a linear rational-expectations model (Blanchard and Kahn, 1980): the response to a known amenity path is unique and bounded, and the two temporal roots of Eq. (13) divide the work between them, as discussed below. The two-region system can be seen as a special

case of the linearized dynamic spatial equilibrium in Kleinman et al. (2023), with one state and one generation per period. In our setting, linearizing at the symmetric steady state makes both roots explicit in the primitive elasticities and weights.

Relative population and value in the local system react differently to news. Inherited relative population is predetermined as the previous generation has already settled. Location values can jump, because they summarize expectations about the entire future amenity path. Boundedness pins down the jump by selecting the value that eliminates the freely growing component associated with λ_u. To see this, define $z_t = \ell_t - \lambda_s \ell_{t-1}$, population beyond propagation at the stable rate. Factoring Q transforms Eq. (12) into

$$z_{t+1} - \lambda_u z_t = -\frac{m}{\rho\psi}\omega_t. \tag{18}$$

Now define the *anticipation factor* $r := \lambda_u^{-1} \in (0,1)$ as the inverse of the unstable root. Appendix B solves Eq. (18) forward to give

$$z_t = \frac{mr}{\rho\psi}\left(\omega_t + r\omega_{t+1} + r^2\omega_{t+2} + \cdots\right) = \frac{m}{\rho\psi}\sum_{j=0}^{\infty} r^{j+1}\omega_{t+j} \tag{19}$$

and $\ell_t = \lambda_s \ell_{t-1} + z_t$. The sum (19) converges for any bounded known amenity path because $0 < r < 1$. The current amenity difference enters with the impact coefficient $mr/(\rho\psi)$, the next generation's at weight r relative to it, and each further generation's with one more factor of r. Population responds today to the entire known future, discounted geometrically at the anticipation factor.

To see the two roots at work, consider a one-period amenity improvement ω at date T, already known at date $t \leq T$. Its contribution to z_t is $m\omega r^{T-t+1}/(\rho\psi)$. Moving the improvement one generation farther into the future multiplies this contribution by r. With a larger r, the weight on future utility declines more slowly with distance into the future, and distant news moves more people today. Before the news arrives, the improvement is not yet part of the known amenity path, so it has no effect on population. Households can only respond to a future change once they know about it.

Once the improvement has passed and the exogenous amenity difference is again zero, $z_t = 0$, and population recovers along $\ell_t = \lambda_s \ell_{t-1}$. The population attracted in anticipation and at implementation fades generation by generation at the rate λ_s, more slowly when λ_s is larger. Under a permanent improvement, the same root governs convergence to the new steady state. The persistence of a steady state is thus a property

of the stable root, which combines population inertia ψ with how population affects utility and how expectations respond to it.

4.3 Different transitions around the same steady state

We now ask how each root is affected by the primitives of the economy, and whether if the stable and unstable roots can move separately. Historical spillovers tie today's utility to past settlement, and patience ties today's choice to descendants' opportunities. Both can support the same long-run geography, but the transition dynamics can be different, telling them apart.

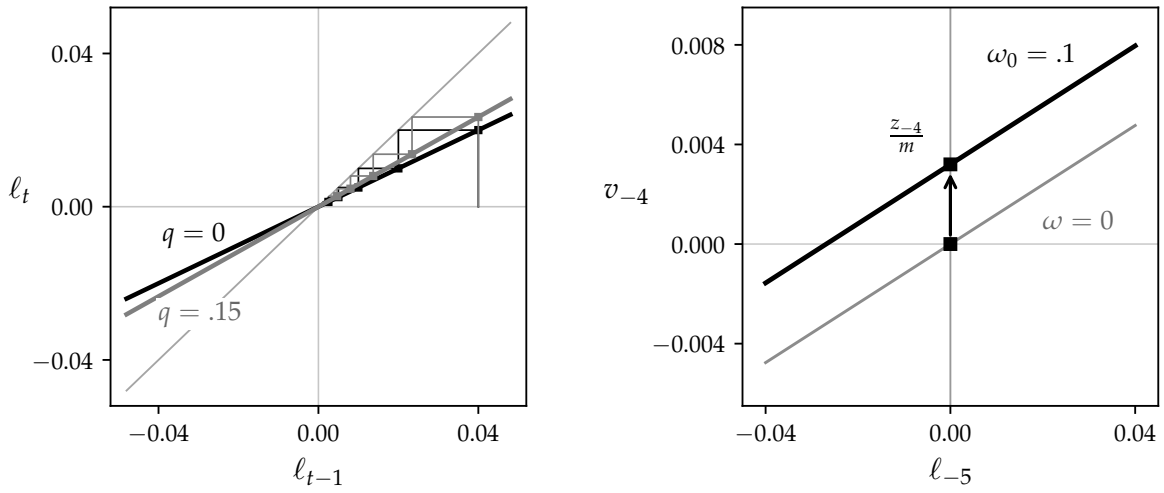
First observe that, at a steady state, $L_{it} = L_{i,t-1} = L_i$, so productivity is proportional to $L_i^{\alpha_1+\alpha_2}$ and amenities to $L_i^{\beta_1+\beta_2}$. Consider moving a part of contemporaneous spillovers from current to previous population sizes, which changes the generation in which settlement affects regional productivity and amenities. When the sums of the elasticities and all other primitives are fixed, the steady-state equations do not change, and neither does the steady-state geography, including asymmetric steady states such as those in Fig. 2. The same holds for the local response to a permanent amenity difference ω around a positive saddle. The within-period equilibrium gives $u^* = G^+\ell^* + \omega$, while migration and location choice require $u^* = \Theta^+\ell^*$. Equating them yields a steady-state relationship:

$$\ell^* = \frac{\omega}{\Theta^+ - G^+}, \tag{20}$$

from which $\omega > 0$ implies $\ell^* > 0$ naturally. This steady-state multiplier depends on the spillovers only through $G^+ = p + q$. The transition roots depend on p and q separately, and this is where timing and patience leave their marks.

Proposition 1 (Timing of spillovers and the effect of patience)**.** *Fix the sums $\alpha_1 + \alpha_2$ and $\beta_1 + \beta_2$ and all other primitives, including ρ. In the two-region economy the set of interior steady states is unchanged by reallocating these elasticities across generations. Locally, consider a transfer h with $p_h = p - h$ and $q_h = q + h$. It preserves $p_h + q_h = p + q$ and the steady-state multiplier. Within the positive-saddle region, increasing h raises λ_s and lowers $r = \lambda_u^{-1}$. Holding (p, q, θ, ψ) fixed instead, an increase in ρ raises r and weakly raises λ_s when $p\psi + q \geq 0$.*

Shifting utility effects from current to inherited population raises λ_s, so population



(a) Persistence at fixed total effects

(b) News at fixed inherited population

Figure 6: Effects of inherited advantages and news.

Note: In both panels, $(\psi, \theta, \rho) = (.4, 1, .8)$. Panel (a): population maps $\ell_{t-1} \mapsto \ell_t$ after a temporary improvement has expired. Black: $(p, q) = (.2, 0)$; gray: $(.05, .15)$, with the same total $p + q = .2$. The slopes are $\lambda_s = .5$ and .59. The staircases follow each economy generation by generation. Panel (b): the location-value difference at $t = -4$ as a function of inherited population, from Eq. (21), in the economy $(p, q) = (.2, 0)$. Gray: no news; black: a one-period amenity improvement $\omega_0 = .1$ at $t = 0$ has just been revealed. The arrow is the value jump due to news, and population moves by m times this amount.

differences recover more slowly. At the same time, it lowers the anticipation factor r, so a given future improvement moves fewer people today, making the economy more persistent in both ways. Greater patience instead raises r, because future opportunities count for more today, and under $p\psi + q \geq 0$ it also weakly raises λ_s. Historical effects and patience can therefore produce similar recovery patterns while moving anticipation in opposite directions.

Figure 6(a) shows the population maps selected by boundedness after a temporary improvement has expired. At a common $G^+ = p + q$ and ρ, the economy with stronger historical effects has the steeper map $\ell_t = \lambda_s \ell_{t-1}$. The staircase steps trace successive generations, and each generation preserves a larger part of the preceding population advantage when λ_s is higher, i.e., in the economy with a greater historical effect.

Before implementation, the same selection has a geometric interpretation in the population–value plane. Combining migration with Eq. (19) gives

$$v_t = \frac{\lambda_s - \psi}{m} \ell_{t-1} + \frac{z_t}{m}. \tag{21}$$

Panel (b) fixes the economy and draws this graph at the announcement date. Revela-

tion leaves the inherited population distribution unchanged, while the future amenity improvement raises z_t and shifts the selected value upward. Current population then rises by m times the value increase through migration, connecting anticipated utility to today's settlement before the actual period-utility improvement arrives.

4.4 History versus expectations

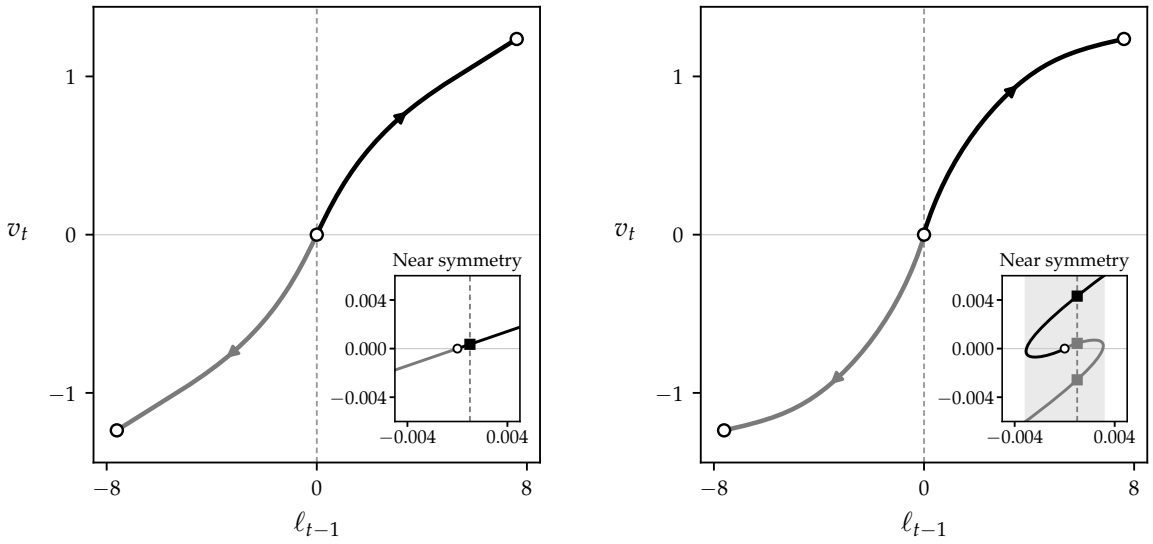
In Fig. 2, the initially larger region became the center. Does initial settlement always decide the destination? The question is familiar from the history-versus-expectations literature (Krugman, 1991a; Matsuyama, 1991; Fukao and Bénabou, 1993). Following Krugman (1991a), I call the set of initial population distributions from which equilibrium paths lead to either of two stable asymmetric steady states the *overlap*; on that set, expectations rather than history select the destination. This section shows that spillover timing changes the overlap while leaving the destinations themselves fixed.

Figure 7 compares two timings of amenity spillovers in the Type L economy of Section 3.5. With stronger historical effects in panel (a), every computed path from a larger region 1 leads to concentration in region 1, and symmetrically for region 2. With weaker historical effects in panel (b), the same steady states remain, but the paths wind around symmetry in Krugman's spiral. The close-up shows paths toward either destination from the same inherited population, supported by different initial location values; the shaded band is the overlap.

The paths wind around symmetry exactly when the temporal roots at symmetry are complex. With $q \geq 0$ and $\chi > \chi^*$, Appendix B.4 shows that this occurs if and only if

$$\left(\sqrt{\psi + mq} - \sqrt{\rho\psi}\right)^2 < m(G^+ - \Theta^+). \tag{22}$$

The left side compares two coefficients of (12): the weight of inherited settlement, $\psi + mq$, which combines population inertia with historical spillovers, and the weight of descendants' opportunities, $\rho\psi$. The right side is m times the net agglomeration force $G^+ - \Theta^+$. When the gap between the two weights is small relative to this force, equilibrium paths near symmetry rotate instead of moving monotonically, and the stable branches of both asymmetric steady states can pass through the same inherited populations. Patient dynasties, weak historical spillovers, and a strong net agglomeration force favor this outcome. The same two coefficients govern the recovery rate and anticipation factor in Proposition 1.



(a) $q = 0.15$ (b) $q = 0.03$

Figure 7: Spillover timing and alternative agglomerated destinations.

Note: Both panels use $\varepsilon = \zeta = \gamma = 1$, $\alpha_1 = \alpha_2 = 0$, and $(\mu, \theta, \rho, b, \chi) = (2, 4, .6, .15, .95)$, where $b = \beta_1 + \beta_2$ and $q = \beta_2$. Only q changes, with $\beta_1 = b - q$. Axes show inherited relative population $\ell_{t-1} = \log(L_{1,t-1}/L_{2,t-1})$ and the location-value difference $v_t = \log(V_{1t}/V_{2t})$. Black paths approach concentration in region 1; gray paths approach concentration in region 2. Arrows indicate forward time; open circles mark steady states. Insets magnify a common neighborhood of symmetry. The dashed line shows $\ell_{-1} = .001$; squares mark initial values of representative paths. The shaded band in (b) marks the overlapping population projections of the computed stable branches.

Appendix B.4 computes how wide the overlap is as primitives change. It widens with patience, and narrows with stronger historical spillovers and greater population inertia: the more today's choice rests on the future, the less the past decides. Immobile demand narrows it sharply. In the numerical example, once the manufacturing expenditure share γ falls even slightly below one, the paths stop rotating and initial settlement again determines the eventual core, even though both asymmetric steady states continue to exist.

The overlap also cannot cover the entire state space. Oyama (2009b) shows, in a setting with a potential function, that the agglomeration pattern the potential favors is absorbing: every path from its neighborhood leads to it, so expectations can override history only away from it. Our economy has no potential, but migration costs supply a similar barrier. The continuation value of either origin mixes both destinations, so expectations can offset at most $\rho \log \mu$ of the current utility difference. Near an asymmetric steady state whose utility advantage exceeds this amount, location values must favor the larger region and relative population cannot change sign, implying that

the other asymmetric steady state is out of reach. Expectations select a destination only from inherited populations close enough to symmetry for the current utility advantage to be small, consistent with Fig. 7(b). Symmetry remains a source throughout, so the multiplicity concerns which center forms and never the path toward a given one. The remaining sections work around convergent steady states, where the inherited population selects a unique path and this multiplicity does not arise.

5 Spatial forces and endogenous geography

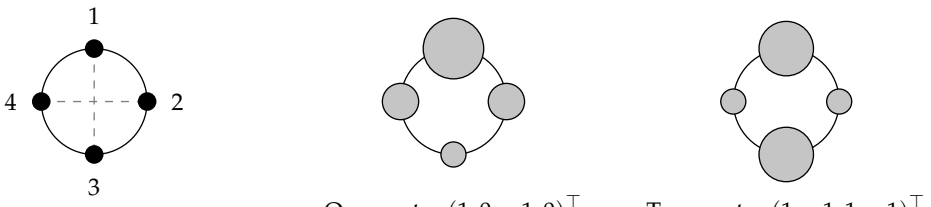
Two regions cannot distinguish a single economic center from several separated ones. Four regions on a ring are the smallest geography that can, and they let us connect the local and global dispersion forces of Section 3.4 to these two spatial patterns. This section asks which pattern concentration favors, which finite configurations each dispersion mechanism supports, and how concern for descendants changes the answer. It extends AMOT's comparison of spatial forces to a dynamic economy with forward-looking location choice.

5.1 Two spatial scales in four regions

Place four otherwise identical regions at equal distances on a ring, as in Fig. 8(a). Trade and migration freeness decline with circular distance: $\tau_{ij}^{-\varepsilon} = \phi_\tau^{d(i,j)}$ and $\mu_{ij}^{-\theta} = \phi_\mu^{d(i,j)}$, with $0 < \phi_\tau, \phi_\mu < 1$ and $d(i,j) = \min\{|i-j|, 4-|i-j|\}$. As in the two-region economy, uniform population distribution is a steady state.

Figure 8(b) shows the two ways population can depart from uniformity. A shift from region 3 toward region 1 creates a one-center pattern, and a shift toward regions 1 and 3 together, away from their neighbors, creates two opposite centers. These are the spatial *eigencomponents*, or *modes*, of the ring, indexed by $k = 1$ for the one-center pattern and $k = 2$ for the two-center pattern. The one-center pattern can be oriented toward any of the four regions, so it spans a two-dimensional space $\mathcal{E}_1$, whereas the two-center pattern spans a line $\mathcal{E}_2$, as defined in Appendix C. Both describe small departures from uniform population, and their combinations need not be finite one-center or two-center configurations and can span all possible forms of deviation.

The two patterns differ in how much of them spatial interaction preserves. In the two-region economy, the single direction of population redistribution $\boldsymbol{e} = (1,-1)^\top$ is



(a) Ring geography (b) Nonuniform spatial patterns

Figure 8: The four-region ring and its two nonuniform spatial patterns.

Note: The solid perimeter in panel (a) connects nearest neighbors; the dashed chords connect opposite regions at circular distance two. In panel (b), gray-disk radius indicates population. The vectors give directions of population change around uniform population. The one-center vector is a representative orientation of the two-dimensional eigenspace $\mathcal{E}_1$; the two-center eigenspace $\mathcal{E}_2$ is one-dimensional.

an eigenvector of the normalized proximity matrix with eigenvalue $(1-\phi)/(1+\phi)$, as in Eq. (5). The ring has two such eigenspaces, and on $\mathcal{E}_k$ the normalized trade and migration matrices again act as scalars, the trade eigenvalue χ_k and the migration eigenvalue ψ_k of pattern k. Writing $\chi = (1-\phi_\tau)/(1+\phi_\tau)$ and $\psi = (1-\phi_\mu)/(1+\phi_\mu)$ for the adjacent-region eigenvalues, we have

$$(\chi_1, \chi_2) = (\chi, \chi^2) \qquad \text{and} \qquad (\psi_1, \psi_2) = (\psi, \psi^2). \tag{23}$$

The two-center pattern alternates between adjacent regions, so averaging across neighbors cancels more of it, whereas the one-center pattern varies slowly around the ring. Both trade and migration therefore preserve more of the coarse pattern: $\chi_1 > \chi_2$ and $\psi_1 > \psi_2$.

Each pattern k then obeys the two-region equations of Section 3 and Section 4 with (χ, ψ) replaced by (χ_k, ψ_k). I write p_k, q_k, and m_k for the resulting coefficients. The spillover elasticities are the same for both patterns. Their responses differ only through the eigenvalues at which the same within-period environment is evaluated as $\mathcal{M}(\chi_k)$.

5.2 Dispersion forces and the ranking of spatial patterns

Which pattern does concentration favor first, and does the answer depend on the dispersion mechanism? The two-region balance of Section 3.3 applies to each pattern separately, with its own trade and migration eigenvalues. For pattern k, compare the

utility advantage that concentration generates with the advantage needed to sustain it:

$$G_k^+ = (\zeta + \alpha_1 + \alpha_2)\mathcal{M}(\chi_k) - 1 + \beta_1 + \beta_2, \tag{24}$$

$$\Theta_k^+ = \frac{1 - \rho\psi_k}{\theta(1 + \psi_k)}. \tag{25}$$

I call $S_k = G_k^+ - \Theta_k^+$ the pattern's *net agglomeration force*: the utility advantage that concentration toward the pattern generates, net of the advantage needed to sustain it.

Assume $q_k \geq 0$ and $S_k < 0$ for $k = 1, 2$, so that the uniform population distribution is a positive saddle in both directions, and thus is stable. In this sense we may also interpret a negative S_k as the *dispersion force* toward pattern k. An exogenous utility profile, like any departure from uniformity, decomposes into the two patterns; fundamentals and the initial spatial distribution determine the weight on each pattern, its *loading*, and its orientation. Applying the same logic as in Eq. (20), a permanent utility profile with unit loading on pattern k yields the steady-state population multiplier

$$\kappa_k = \frac{1}{\Theta_k^+ - G_k^+}. \tag{26}$$

A larger net agglomeration force (or, smaller dispersion force) means stronger amplification of a given loading. It also means that the pattern reaches its persistent-agglomeration boundary $S_k = 0$ sooner when the current amenity elasticity rises by a common amount, since such an increase raises every net agglomeration force equally while other primitives stay fixed.

At the first crossing, the uniform distribution stops being a positive saddle in that pattern's direction and becomes a source there, as symmetry did beyond χ^* in Section 3.5. Concentration in that pattern can then sustain itself, and nonuniform steady states emerge in the multi-region economy as they did with two regions. Section 5.3 below shows the configurations they form. I call this crossing the onset of endogenous agglomeration and the experiment the *common-elasticity onset experiment*. The pattern with the largest net agglomeration force becomes critical first.

Proposition 2 (Spatial structure and amplification). *Suppose $\zeta + \alpha_1 + \alpha_2 > 0$ and fix a common dynastic weight $\rho > 0$.*

(a) The all-mobile economy with $\gamma = 1$ satisfies $S_1 > S_2$. The one-center component therefore has the larger steady-state multiplier on $S_k < 0$ and becomes critical first in the common-elasticity onset experiment.

(b) *With immobile demand,* $\gamma < 1$ *and* $\gamma < 2\varepsilon$*, the reverse ordering* $S_2 > S_1$ *holds, by definition, if and only if* $G_2^+ - G_1^+ > \Theta_2^+ - \Theta_1^+$*, where*

$$\Theta_2^+ - \Theta_1^+ = \frac{(1+\rho)(\psi - \psi^2)}{\theta(1+\psi)(1+\psi^2)} > 0. \tag{27}$$

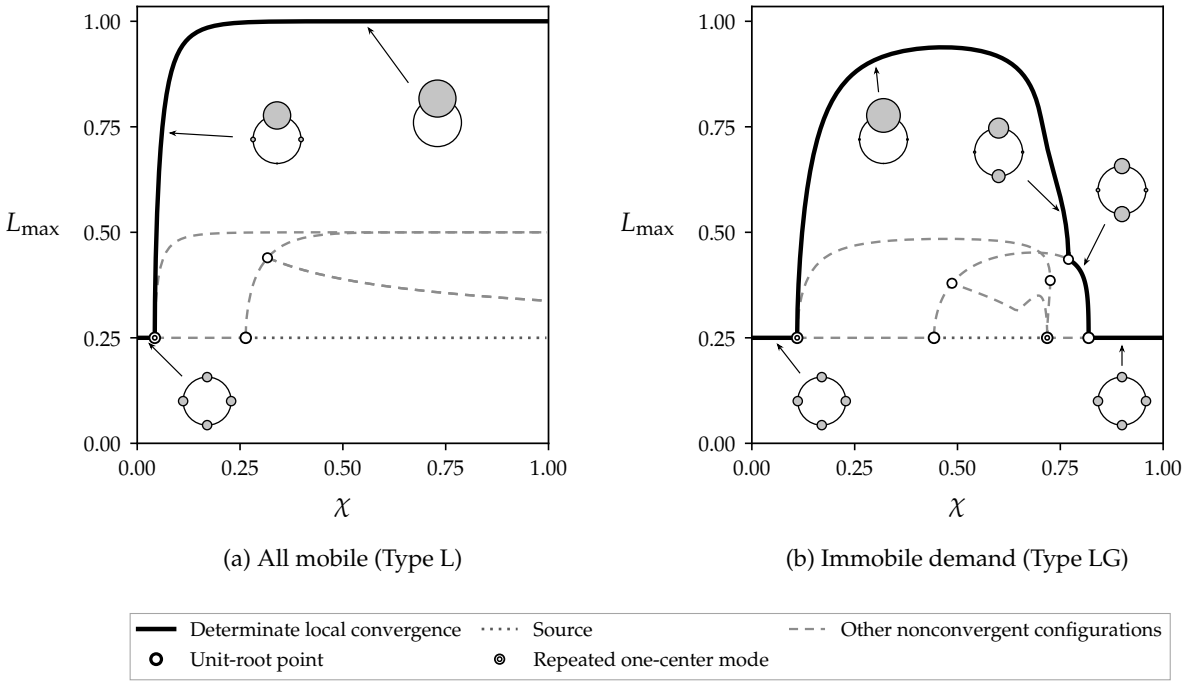
The inequality in part (b) compares two effects. Its left side is the two-center pattern's advantage in the utility effect that concentration generates. Its right side is the two-center pattern's *disadvantage* in sustaining itself. Migration dynamics preserve less of the finer pattern, since $\psi_2 = \psi^2 < \psi = \psi_1$. Before any comparison of goods-market forces, the one-center pattern therefore has the lower threshold, meaning that a reversal as in part (b) needs a sufficiently strong within-period force to cover this gap.

The goods-market structure decides whether such a force exists. In the all-mobile Type L economy, $\mathcal{M}$ increases in χ, so $\mathcal{M}(\chi_1) = \mathcal{M}(\chi) > \mathcal{M}(\chi^2) = \mathcal{M}(\chi_2)$, whereby the one-center pattern has both the larger utility effect and the lower threshold. With immobile demand, the single-peaked response permits the opposite ranking: Appendix A.3 shows that there is a unique $\chi^\dagger$ above which the two-center utility effect is larger. Whether that reversal is large enough to change the ranking of net forces depends on whether it exceeds the threshold gap in Eq. (27). A model's long-run geography thus reflects both the reach of its dispersion force and how migration maintains population patterns.

5.3 Endogenous spatial patterns

Proposition 2 identifies the directions in which concentration can reinforce itself. To see the patterns these forces support, I solve the original nonlinear steady-state equations and, as in AMOT, vary trade costs while holding geography, migration technology, spillovers, and the dynastic weight fixed. The question is how the number and spacing of centers change as regions become more closely connected.

Figure 9 compares an all-mobile economy (i.e., Type L) with one containing immobile agricultural demand (i.e., Type LG). The two economy share every primitive except the manufacturing expenditure share and the associated agricultural endowment. The vertical axis is the population share of the largest region, $L_{\max} := \max_i L_i$. Solid black branches are stable steady states to which nearby initial population distributions converge along a unique dynamic path. Gray branches are steady states without this



(a) All mobile (Type L)

(b) Immobile demand (Type LG)

Figure 9: Dispersion mechanisms and endogenous steady-state spatial patterns.

Note: The horizontal axis χ rises with trade costs; the vertical axis is the largest regional population share, $L_{\max} = \max_i L_i$. Panels (a) and (b) use $\gamma = 1$ and .3, respectively. Both use $\rho = .8$, $(\varepsilon, \zeta, \theta, \psi) = (5, .8, 20, .5)$, and $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (.2, .1, -.4, .3)$, with uniform fundamentals. Agricultural labor relative to each region's uniform mobile population is zero in panel (a) and 7/3 in panel (b). In the insets, gray-disk diameter is proportional to population share on a common scale.

property and deemed unstable.

In the all-mobile economy, the convergent configurations have a single peak throughout. At high trade costs almost all mobile workers live in one region. As trade becomes cheaper, the terms-of-trade loss from concentration strengthens, and population spreads away from the center until the branch meets uniform population.

With immobile demand, high trade costs keep production near consumers in all four regions, so uniform population is the convergent configuration. As integration makes serving consumers from fewer places cheaper, the convergent configuration changes to two equal opposite centers, then to two unequal centers, and finally to a single peak, before returning to uniform population at low trade costs. The insets separate changes in the number of centers from changes in concentration around one of them: at $\chi = .80$ the two peaks are equal, at .76 they are unequal, and at .30 the opposite region holds fewer residents than either neighbor.

These branches are computed over the displayed trade-cost range, and they vary trade costs rather than the common amenity elasticity of Proposition 2. What they

show is that the agricultural-sector dispersion force generated by immobile consumers changes which spatial patterns, and hence which regional reallocations, a model can support in the long run.

5.4 Forward-looking choice and spatial patterns

Patience lowers the two-region threshold through $\rho\psi$. With two patterns, does it favor one of them? At given within-period utility effects and migration shares, each net agglomeration force rises linearly in ρ with slope $\psi_k/[\theta(1+\psi_k)]$, which is larger when migration frictions preserve more of the pattern. On the ring, the one-center pattern has the larger migration eigenvalue, so descendants' opportunities give it the greater additional advantage. The threshold gap in Eq. (27) rises with ρ, so greater patience makes a two-center reversal harder, as summarized below.

Proposition 3 (Migration eigenvalues and the effect of patience)**.** *Hold within-period utility effects and migration shares fixed.*

(a) An increase in ρ can switch the first critical component in the common-elasticity onset experiment only toward components with weakly larger migration eigenvalues. On the four-region ring, this favors the one-center component.

(b) Where both components are positive saddles, greater patience increases both steady-state multipliers. It raises the two-center multiplier proportionally more if and only if $\kappa_2/\kappa_1 > b_1/b_2$, where $b_k = \psi_k/[\theta(1+\psi_k)]$ is the slope of S_k in ρ. This holds on an open set of parameters with immobile demand and never in the all-mobile economy.

Figure 10 illustrates the threshold effect at low trade costs. Within each row, all primitives except patience are fixed. As trade costs vanish, the utility effect of concentration approaches .08 for both patterns. This is below both thresholds under myopia and above both at $\rho = .8$. Uniform population therefore permits convergence under myopia and becomes a source under greater patience, even with nearly frictionless trade (Appendix C.3).

The uniform population distribution remains a steady state in every column but the stability properties are different across them. In the middle column, at $\rho = .4$, the one-center component has already lost convergence at low positive trade costs while the two-center component still has a stable temporal root, in line with Proposition 3. Immobile demand continues to anchor activity at high trade costs, where the economy in the bottom row regains convergence around uniform population. Where convergence

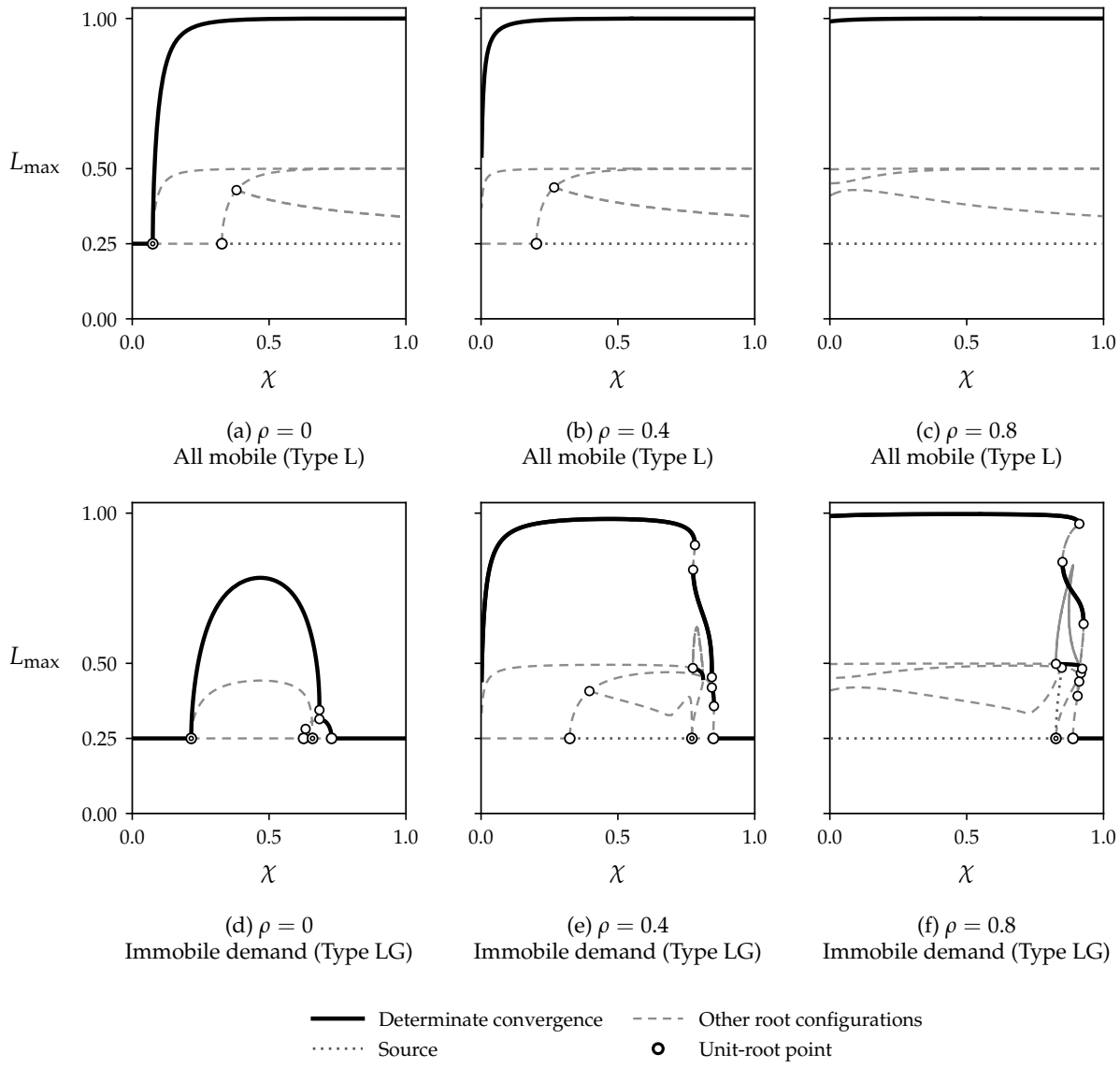


Figure 10: Forward-looking choice and nonconvergence at low trade costs.

Note: The bottom row has $\gamma = .3$. The middle column is the boundary case in which the one-center threshold equals the low-trade-cost utility effect .08. All panels hold $(\varepsilon, \zeta, \theta, \psi) = (5, .8, 5, .75)$ and $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (.2, .1, -.32, .3)$ fixed; agricultural endowments and uniform fundamentals follow Fig. 9. Double circles mark the repeated one-center direction at symmetry.

remains possible, patience raises both steady-state multipliers. Their relative increase need not follow the onset ranking: a two-center response already close to its boundary can be the more sensitive one.

6 The geography of anticipated change

We next leave the economy unchanged and vary only when households learn of an improvement to isolate earlier information from the weight they place on descendants. As the theoretical experiment, we consider the improvement in regional living conditions described in the introduction.

A permanent exogenous log-amenity profile $\boldsymbol{\omega}$ is implemented in period T and first learned in period $T - H$, where the integer $H \geq 0$ is the notice horizon. If $H = 0$, implementation is a surprise. From revelation onward, the path is perfectly foreseen. I measure its population effect relative to the no-improvement equilibrium from the same initial spatial distribution. Since a period is a generation, even one period of notice represents information that arrives well before implementation. Interpreting a period as a decade shortens the implied calendar horizons without changing the analysis.

With the amenity profile and implementation date given, we first ask how earlier revelation changes the spatial composition of settlement by implementation, then how that composition determines which regions gain population. The responses are local around uniform population, with both patterns positive saddles ($q_k \geq 0$, $S_k < 0$) and zero population response before revelation. Write $\Delta\boldsymbol{\ell}_t = (\Delta\ell_{it})$ for the vector of log-population responses along the path.

6.1 Earlier notice and spatial composition

Earlier notice of an improvement changes not only how much people move before it arrives but also where they move. To isolate the role of information, hold every structural parameter, the realized amenity profile, and its implementation date T fixed, and vary only the notice horizon H. Write $r_k = \lambda_{u,k}^{-1}$ for component k's anticipation factor and define

$$K_k := \kappa_k(1 - \lambda_{s,k}), \qquad \xi_k := \lambda_{s,k} r_k, \qquad \text{and} \qquad B_H(x) := \sum_{j=0}^{H} x^j. \tag{28}$$

Here, K_k represents the initial population impact of a surprise per unit loading. With one period of notice, households already move at $T-1$. The movement newly induced at that date, the object z_t of Section 4.2, is $K_k r_k$ per unit loading, and the fraction $\lambda_{s,k}$ of it propagates to T. Its contribution at implementation is therefore $K_k \xi_k$. Each additional period of notice adds another power of ξ_k, combining the decay of anticipation with subsequent population propagation. This gives the geometric sum $B_H(\xi_k)$. Let $\mathbf{\Pi}_k$ denote the orthogonal projector onto $\mathcal{E}_k$, so that $\mathbf{\Pi}_k \boldsymbol{\omega}$ is the component of a profile on pattern k. We have the following characterization.

Proposition 4 (Notice and spatial reweighting). *Suppose both components are positive saddles. A permanent profile $\boldsymbol{\omega}$ is revealed at $T-H$ and implemented at T, with a zero population response at $T-H-1$. Its implementation response is*

$$\Delta \boldsymbol{\ell}_T = \sum_{k=1}^{2} K_k B_H(\xi_k) \mathbf{\Pi}_k \boldsymbol{\omega}. \tag{29}$$

(a) When both patterns carry nonzero loadings, the ratio of the one-center to the two-center response magnitude at implementation, divided by the same ratio under surprise, equals $B_H(\xi_1)/B_H(\xi_2)$. It rises with H when $\xi_1 > \xi_2$, falls when $\xi_1 < \xi_2$, and is constant at equality.

(b) In a Type L economy with $\zeta + \alpha_1 > 0$, $\alpha_2 \geq 0$, and $q_2 \geq 0$, we have $\xi_1 > \xi_2$. A Type LG economy permits $\xi_2 > \xi_1$ on an open set of parameters with $q > 0$.

Equation (29) consists of three separate components: the initial impact K_k, the accumulation $B_H(\xi_k)$ over the notice period, and the part $\mathbf{\Pi}_k \boldsymbol{\omega}$ of the improvement that falls on each pattern. Earlier notice raises the response at implementation of every pattern the improvement touches, and the pattern with the larger ξ_k gains proportionally more.

At a particular region, the two patterns' contributions can have opposite signs, as the example below illustrates, so the region's net response need not rise with notice. Which pattern dominates a region's response also depends on the initial impacts and on how the improvement is spread across the ring, as formally discussed in Appendix D.

In the all-mobile Type L economy, the stated sign restrictions give the one-center component both slower recovery and a larger anticipation factor, so it accumulates the larger response. Immobile demand in a Type LG economy allows the utility effects to favor the two-center component strongly enough to reverse the ordering of ξ_1 and ξ_2.

Figure 11 uses a Type LG economy at high trade costs, where $\xi_1 < \xi_2$. Serving

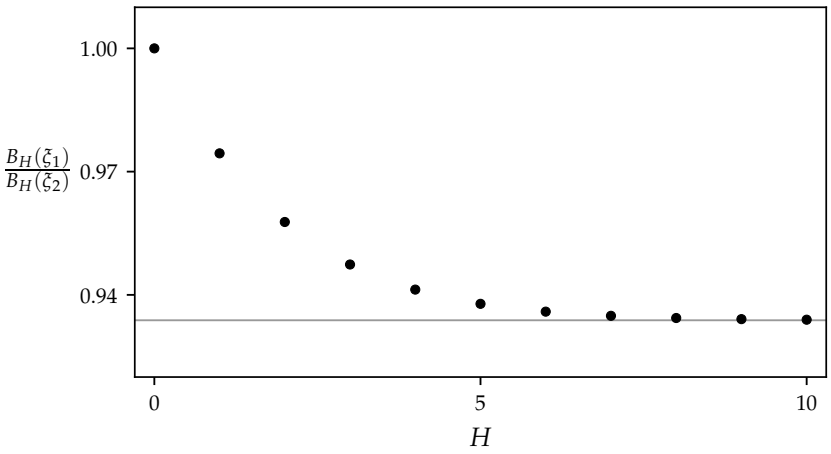


Figure 11: Earlier notice can shift the response toward the two-center pattern.

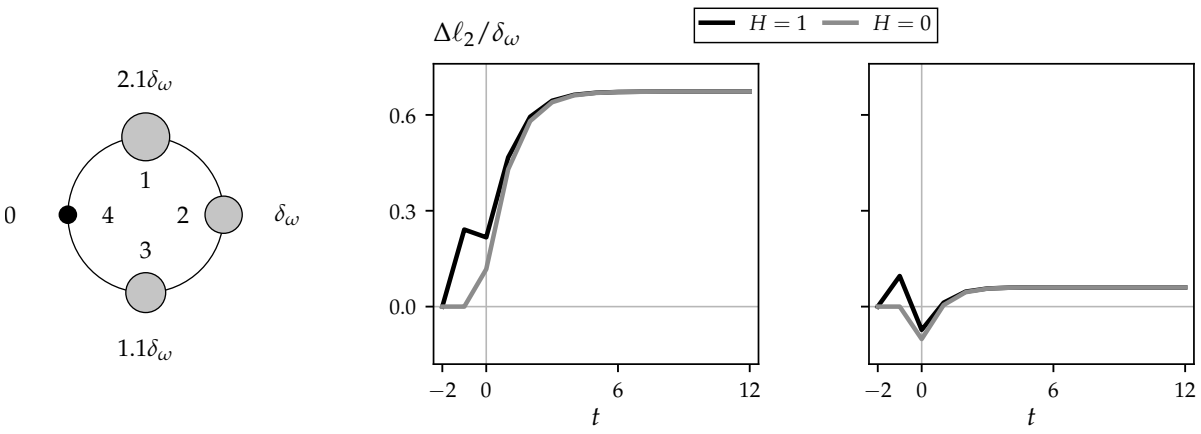
Note: The economy, permanent amenity profile, and implementation date are fixed; only the notice horizon H varies. The vertical axis is the ratio $B_H(\xi_1)/B_H(\xi_2)$, which equals the ratio of the one-center to the two-center response at implementation divided by the same ratio under surprise, for $(\varepsilon, \zeta, \gamma, \chi, \psi, \theta, \rho) = (5, .8, .4, .9, .7, 3, .8)$ and $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (.5, .02, -.25, .01)$. Both patterns carry nonzero loadings. Filled black circles show the integer horizons $H = 0, \ldots, 10$; the thin gray horizontal line is the limit as $H \to \infty$. This economy has immobile demand and positive historical utility effects in both components.

immobile consumers in every region from a single center is costly there, so two opposite centers generate the larger utility advantage, and the product $\xi_2 = \lambda_{s,2} r_2$ is larger for two centers even though migration preserves less of the finer pattern. The ratio in the figure falls below one with any notice, keeps falling as the horizon lengthens, and levels off. Earlier notice raises the two-center pattern's share of the response, while the two-center response need not become larger in absolute terms.

The contrast with patience is instructive. Greater patience in Section 5.4 moves the agglomeration boundary. Earlier notice moves no force or threshold but it adds anticipatory moves whose effects survive until implementation, so its spatial effect runs through $\lambda_{s,k} r_k$ alone. Patience can favor the coarse pattern at onset while notice favors the finer pattern, because the two act on different margins of intergenerational adjustment.

6.2 Regional winners and losers under the same improvement

The previous subsection compared the responses of the two patterns. The population change predicted for any single region adds the contributions of both patterns, so the comparison alone does not say which regions gain. This subsection shows, with one example, that the same improvement can make a region a winner in a Type L economy



(a) Amenity profile (b) $\gamma = 1$ (Type L) (c) $\gamma = .4$ (Type LG)

Figure 12: Same improvement and opposite population effects at implementation.

Note: Panels (b)–(c) show region 2's log-population response to the amenity profile in panel (a), with implementation at $t = 0$; panel (b) is the all-mobile economy and panel (c) the economy with immobile demand. Both use $(\varepsilon, \zeta, \chi, \psi, \theta, \rho) = (5, .8, .8, .3, 3, .8)$ and $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (.1, .02, -.15, .01)$; agricultural labor per region is zero and 1.5 in reference units with mobile population one per region.

and a loser in a Type LG economy.

Consider an announced amenity improvement concentrated in regions 1–3, with no direct gain in region 4, as in Figure 12(a): $\boldsymbol{\omega} = \delta_\omega(2.1, 1, 1.1, 0)^\top$ with $\delta_\omega = 10^{-3}$. Region 2 gains directly, but its neighbors 1 and 3 gain more. Whether households move into or out of region 2 depends on how they weigh its own gain against the larger gains next door, together with the population and utility changes these set off.

Projecting $\boldsymbol{\omega}$ onto the eigenspaces of Section 5.1 yields

$$\boldsymbol{\Pi}_1\boldsymbol{\omega} = \frac{\delta_\omega}{2}(1, 1, -1, -1)^\top \qquad \text{and} \qquad \boldsymbol{\Pi}_2\boldsymbol{\omega} = \frac{1.1\delta_\omega}{2}(1, -1, 1, -1)^\top. \tag{30}$$

The uniform component raises all amenities equally and leaves relative population unchanged. The one-center component sets the improving side of the ring, regions 1 and 2, against the far side, and favors region 2. The two-center component sets regions 1 and 3 against regions 2 and 4; on this comparison, region 2 is the weaker of the three improving regions, and the component draws population away from it. Its loading, 1.1 against 1, is also the larger. Which of the two effects wins depends on how strongly the economy amplifies each pattern.

Panels (b) and (c) answer this for two economies that differ only in the manufacturing expenditure share and the associated agricultural endowment. In the all-mobile Type L economy, the one-center pattern is amplified more, and region 2 gains population at

implementation and in the long run. In the Type LG economy with immobile demand, the two-center pattern is amplified more: region 2 loses population at implementation and recovers only a small long-run gain. With one period of notice, region 2 in the Type LG economy even gains slightly before implementation and then loses, because the one-center pattern responds more strongly to news while the two-center pattern dominates once the improvement arrives.

The opposite signs do not come from notice, as we see the gray surprise paths already differ in sign. Nor do they follow from the ranking of the two patterns. The net agglomeration force S_k favors the one-center pattern in both economies in this example, yet with immobile demand the two patterns' initial impacts are nearly equal, and the larger two-center loading decides. A region's gain or loss is therefore the sum of the two terms in Eq. (29), and the composition of dispersion forces enters each of them. With the theoretical results in place, the remaining question is practical: what must a quantitative model get right to predict such regional gains and losses, and what evidence can show that it does?

7 What evidence validates anticipated adjustment?

The regional effect of an announced improvement depends on how a model combines two spatial responses, each shaped by its dispersion forces, inherited settlement, and the weight on descendants, so a dynamic QSM must get this balance right. In practice, such models are validated on population paths after surprise changes, the episodes the data contain; Adão et al. (2025) show how to test a model's causal prediction for such an observed change when other shocks also move the data. This section asks what those paths pin down about an announced change. They fix how far population eventually moves and how fast it recovers, but not how much of the move happens before the change arrives, so the sign of a region's response to an announcement can remain open. I then ask which additional observations close this gap.

7.1 Equal surprise paths under conflicting announcements

The natural validation target is adjustment after an unanticipated change, so we first ask what such a path identifies. Suppose a permanent amenity improvement is implemented without advance notice at T, with a zero population response at $T-1$. Component k's

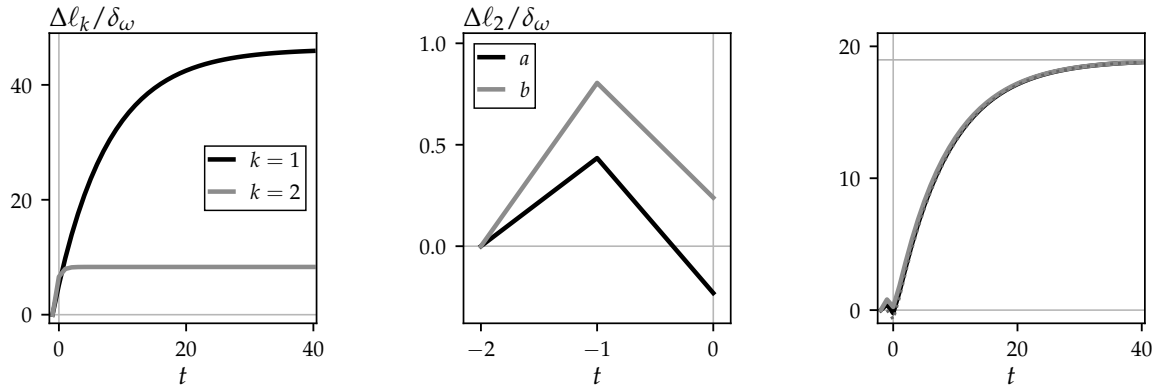
entire local population path is

$$\Delta \ell_{k,T+h} = \kappa_k \cdot \left(1 - \lambda_{s,k}^{h+1}\right) \omega_k \qquad \text{where} \qquad h \geq 0. \tag{31}$$

The path is a curve with two parameters: where it ends, set by the steady-state multiplier κ_k, and how fast it gets there, set by $\lambda_{s,k}$. With a known utility loading ω_k, the whole path delivers these two objects and nothing else. In particular, it says nothing about the anticipation factor r_k, because nobody anticipated anything. Observing more dates after the same surprise sharpens the estimates of the multiplier and the recovery rate while leaving anticipation untouched.

One might hope that requiring the same spillover elasticities across the two spatial patterns closes this gap. It does not. Appendix E constructs a four-region family with common trade and migration geography, goods-market shares, and migration technology, in which patience and the four spillover elasticities move together so that both pairs $(\kappa_k, \lambda_{s,k})$ stay fixed. Every member of the family therefore produces the same local population path after every permanent surprise, whatever its spatial profile, while the anticipation factors vary along the family. The match covers local population paths only. Thus, wage and value responses along a transition, and nonlinear paths, are unmatched, and may or may not tell the members apart.

Figure 13 shows what this means for a common nonnegative amenity profile. The surprise paths coincide, yet one period of notice produces a population loss in region 2 in economy a and a gain in economy b. As in Section 6, the two patterns pull region 2 in opposite directions. The economies differ only in how much anticipatory movement each pattern accumulates, $B_H(\xi_k)$, and that difference is enough to flip the sign at implementation. Both economies approach the same long-run response, because the analysis is local around a convergent uniform steady state and the matching fixes the multipliers κ_k that set this common limit. In the computed examples, the trade-cost range in which the symmetric or uniform steady state is convergent is also the range in which it is the only steady state (cf. Figs. 2 and 9). Local convergence here thus plays the role that global uniqueness conditions play in static QSMs (Allen and Arkolakis, 2014; Kucheryavyy et al., 2024): the counterfactual response is unique, and the question is not which steady state the economy reaches but how it gets there. The same question at a stable asymmetric steady state, such as the agglomerated configurations of Section 5.3, remains open.



(a) Permanent surprise (b) Region 2 (implementation) (c) Region 2 (common limit)

Figure 13: A fit of permanent-surprise paths can leave regional anticipation unresolved.

Note: Implementation is at $t = 0$; responses are per unit of the amenity amplitude δ_ω. Panel (a): surprise paths of both components under the permanent profile $\delta_\omega(4.2, 1, 3.2, 0)^\top$, identical across economies. Panels (b)–(c): region 2 with one period of notice in economies a and b; the dotted curve in (c) is the common surprise path. Both are Type L economies with $(\varepsilon, \zeta, \gamma, \chi, \psi, \theta) = (3, .3, 1, .4, .43, 3)$; patience rises from $\rho_a = .3$ to $\rho_b = .85$ with the four spillover elasticities adjusted to preserve the surprise responses.

The matched family differs from the incidence example of Section 6 in what it holds fixed. There, dispersion mechanisms differed at common patience and spillovers, and the surprise paths differed with them. Here, patience and spillovers move together to keep every surprise path the same. The lesson is a validation problem, in which an exact fit of dynamic local population paths after permanent surprises can leave the sign of an announced regional response undetermined.

7.2 Information about both sides of a change

An announced event supplies the missing information directly, because it is observed on both sides of implementation. Assume a permanent improvement with no other shocks in the event window, known revelation and implementation dates, and a nonzero loading on component k. Using the stable root from the surprise path, define the newly induced movement, the population response net of what the previous period's response propagates,

$$z_{k,t} = \Delta\ell_{k,t} - \lambda_{s,k}\Delta\ell_{k,t-1}.$$

Then, for an improvement announced by $T-1$, we recover

$$\frac{z_{k,T-1}}{z_{k,T}} = r_k. \tag{32}$$

The ratio compares the movement newly induced one generation before implementation with the movement newly induced at implementation. It is the weight households place on a change one generation away, and the size of the improvement cancels. I call it the *news ratio*. Together with the fitted κ_k and $\lambda_{s,k}$, it fixes component k's response at every notice horizon in Eq. (29). While we here assume there are no other shocks, with other shocks in the event window, each newly induced movement must first be isolated as a causal response to the announcement, which requires exogenous variation in regional exposure to it of the kind Adão et al. (2025) use to test causal predictions.

A change of known duration supplies the same information without any observation before implementation. A one-period surprise that everyone knows will expire moves fewer people than a permanent one of the same size, because descendants will not receive the gain. As discussed, its impact is $K_k(1-r_k)$ per unit loading rather than K_k. The shortfall measures r_k. A fit of permanent-surprise paths leaves this statistic free, since it does not imply a fit of responses to shocks of every duration.

Information from outside the event window can also help. With known migration parameters, specifying patience selects a single member of the matched family. More regions help as well. With three or more spatial patterns, the requirement that all of them share the same spillover elasticities is a genuine restriction, and when their goods-market and migration responses vary independently enough it restricts patience and spillover timing. Observing more regions adds spatial information, whereas observing more dates of an already matched surprise path does not.

7.3 Validating a regional prediction

Even when the structure remains imperfectly known, validation can target the regional prediction itself, as Adão et al. (2025) argue for causal predictions in general. Suppose the admissible members of the matched family are those with patience in a given compact interval. For the proposed profile and notice horizon, compute the smallest and largest implementation responses of the region across those members. Every value in between is attained within the family, so the two extremes bound the prediction, as discussed in Appendix E.5.

In the example of Fig. 13, $\rho \in [.3, .85]$ gives region 2 an implementation response between $-.23$ and $.24$ per unit amplitude with one period of notice, so both a loss and a gain remain possible. Information that restricts the one-center news ratio to the range generated by $\rho \in [.7, .8]$ narrows the prediction to $[.14, .21]$: every remaining economy predicts a gain, although patience and spillovers still vary. These bounds are deterministic and conditional on the assumed model family, and describe what the admissible economies allow.

Restrictions on primitives work the same way. Economy b requires a negative historical amenity elasticity. Imposing $\beta_2 \geq 0$ on the model family confines patience to approximately $[.30, .35]$, where region 2's response lies in $[-.23, -.17]$, so a loss becomes certain. Where such a restriction is credible, it substitutes in part for the news ratio; where it is not, the news ratio or a known-duration impact supplies what is missing.

The role of news ratios and known-duration impacts in validation is thus direct. Even an interval for either statistic can rule out economies that fit the permanent surprise paths yet disagree about the anticipated regional sign. If the prediction interval still straddles zero, those observations do not settle the sign, and more is needed. What must be observed depends on which regional counterfactual is being reported.

7.4 Heterogeneous steady states

The symmetric ring made the matched model family exact, but symmetry is not what causes the validation problem. At any steady state, permanent-surprise paths pin down how population propagates and how far it eventually moves, whereas announcement responses also depend on how anticipated future utility enters current location choice through continuation values. What symmetry adds is transparency. In the symmetric economy, a small improvement loads onto fixed spatial response directions. At a heterogeneous steady state, endogenous variables such as wages, trade shares, and migration shares differ across regions and the response directions change with them in a nontrivial manner. Appendix F develops the exact local matrices for that case, keeping the distinction between different intergenerational linkages in our model while letting their spatial directions interact.

The disagreement survives small departures from symmetry. Appendix F proves that a strict regional sign disagreement persists under sufficiently small perturbations of a regular matched economy, with the surprise responses remaining approximately

matched, and reports an example in which it persists under a small perturbation but not under a larger one. The asymmetric examples in AMOT and the 432-region Japanese application of Sugimoto, Takayama and Takagi (2025) point the same way: a model's spatial response structure matters in irregular many-region counterfactuals. Together, these results suggest that the validation problem is relevant for QSMs, without establishing that announcement responses reverse in a calibrated heterogeneous economy. Substantial heterogeneity requires computing the response operators at the actual steady state and checking which common observations restrict them; in a later version of this paper, I plan to carry out this comparison numerically on the Japanese regional system, using observed trade and migration flows.

8 Conclusion

The spatial reach of dispersion forces restricts the geography a model can generate, as AMOT show. This paper asks what that restriction implies once location choice is linked across generations through multiple channels. As expected, the two interact to shape the geography of adjustment. The dispersion mechanism decides which spatial patterns are favored, and the intergenerational linkages decide how strongly each pattern is amplified, how long it persists, and how far ahead households respond to it.

Patience and notice act on different margins. Greater patience lowers the agglomeration threshold most for the patterns that migration preserves, so on the ring it favors a mono-centric pattern at the onset of endogenous agglomeration. Earlier notice changes no force or threshold but reweights the response toward the pattern that accumulates more anticipatory movement, which with immobile demand can be the two-center pattern. The same improvement can therefore make a region a winner under one dispersion mechanism and a loser under another.

For dynamic spatial counterfactuals, the practical lesson is that fitting population paths after permanent surprises does not validate a model's response to an announced change, whereas news ratios, changes of known duration, and spatial or primitive restrictions can. Which region gains from an announced improvement depends on the reach of the dispersion mechanism and on how anticipation enters location choice. Evaluating such an improvement therefore requires evidence on both, beyond the population paths a model already fits.

A Within-period equilibrium

A.1 Equilibrium conditions

On τ_{ij}, the first index denotes destination and the second denotes origin. With manufacturing labor income $Y_{it} = w_{it}L_{it}$ and agricultural income $R_{it}^{\mathrm{A}} = w_t^{\mathrm{A}}\bar{L}_i^{\mathrm{A}}$, the KLR goods-market conditions are

$$P_{jt}^{-\varepsilon} = \sum_i \tau_{ji}^{-\varepsilon} a_{it}^{\varepsilon} w_{it}^{-\zeta\varepsilon} P_{it}^{-(1-\zeta)\varepsilon}, \tag{A.1}$$

$$\frac{Y_{it}}{\zeta} = \sum_j \vartheta_{ji,t}\left[\gamma(Y_{jt} + R_{jt}^{\mathrm{A}}) + \frac{1-\zeta}{\zeta}Y_{jt}\right], \quad \vartheta_{ji,t} = \frac{\tau_{ji}^{-\varepsilon} a_{it}^{\varepsilon} w_{it}^{-\zeta\varepsilon} P_{it}^{-(1-\zeta)\varepsilon}}{P_{jt}^{-\varepsilon}}, \tag{A.2}$$

$$\sum_i R_{it}^{\mathrm{A}} = (1-\gamma)\sum_i (Y_{it} + R_{it}^{\mathrm{A}}). \tag{A.3}$$

With agriculture, its wage w_t^{A} is the numeraire. One manufacturing market equation is redundant given the others and global agricultural clearing. Without agriculture, we omit Eq. (A.3) and use $\sum_i Y_{it} = 1$ to fix the nominal scale. The utility and migration relations are given in Eqs. (1) to (4).

Local differentiation is conditional on a positive solution whose reduced market Jacobian, after normalization, is nonsingular. The implicit function theorem then supplies a differentiable local utility map. At symmetry, the calculation below has a strictly positive determinant, and aggregate normalization removes the remaining indeterminacy along the uniform direction $\mathbf{1} = (1, \ldots, 1)^\top$.

A.2 General equilibrium multiplier

Consider a symmetric trade share matrix with a real nonuniform eigenvalue $x \in [0, 1)$, i.e., χ or χ_k in the main text. All expressions in this subsection are first-order log deviations in the associated eigencomponent.

Global agricultural clearing implies $\sum_i R_i^{\mathrm{A}} = (1-\gamma)\sum_i Y_i/\gamma$. At symmetry, agricultural income has no nonuniform response. The bracketed expenditure in Eq. (A.2), normalized by its symmetric value Y/ζ, therefore responds to manufacturing income with coefficient $\eta := 1 - \zeta(1-\gamma)$. This equals the intermediate expenditure share plus final manufacturing expenditure out of manufacturing labor income.

Define $B(x) = 1 - \eta x$ and $D(x) = 1 - (1-\zeta)x$. Differentiating the price index and the market-clearing equation gives

$$D(x)\widehat{P} = x(\zeta\widehat{w} - \widehat{a}) \qquad \text{and} \qquad B(x)\widehat{Y} = \varepsilon(\widehat{a} - \zeta\widehat{w} + (x - 1 + \zeta)\widehat{P}), \tag{A.4}$$

where, for any variable X_{it}, we write $\widehat{X}_{it} = \log(X_{it}/\overline{X})$ for its log deviation from the symmetric value. Time subscripts are omitted here. For the second identity, we note that differentiating a trade share gives $\widehat{\vartheta}_{ji} = \varepsilon[\widehat{a}_i - \zeta\widehat{w}_i - (1-\zeta)\widehat{P}_i + \widehat{P}_j]$.

From $Y_{it} = w_{it}L_{it}$, we have $\widehat{Y} = \widehat{w} + \widehat{L}$, so that $\widehat{w} = \widehat{Y} - \widehat{L}$. At fixed regional income, additional workers reduce income per worker. Define the composite cost shifter $\widehat{X} = \widehat{a} + \zeta\widehat{L}$. Then, Eq. (A.4) become

$$D(x)\widehat{P} = x(\zeta\widehat{Y} - \widehat{X}) \qquad \text{and} \qquad [B(x) + \varepsilon\zeta]\widehat{Y} - \varepsilon(x - 1 + \zeta)\widehat{P} = \varepsilon\widehat{X}. \tag{A.5}$$

Their determinant is $\Delta(x) := D(x)B(x) + \varepsilon\zeta(1-x^2) > 0$, because $B, D > 0$ and $x < 1$. Cramer's rule, also using $D(x) - x(x-1+\zeta) = 1 - x^2$ and $D(x) - \gamma\zeta x = B(x)$, yields

$$\widehat{Y} = \frac{\varepsilon(1-x^2)}{\Delta(x)}\widehat{X} \qquad \text{and} \qquad \widehat{Y} - \gamma\widehat{P} = \mathcal{M}(x)\widehat{X}, \tag{A.6}$$

where $\mathcal{M}$ is the general equilibrium multiplier of the form

$$\mathcal{M}(x) := \frac{\varepsilon(1-x^2) + \gamma x B(x)}{\Delta(x)} > 0. \tag{A.7}$$

As we see, $\mathcal{M}$ translates the induced composite cost deviation $\widehat{X} = \widehat{a} + \zeta\widehat{L}$ to the deviation of real regional income $\widehat{Y} - \gamma\widehat{P}$. Consequently, we obtain

$$\widehat{u} = \mathcal{M}(x)(\widehat{a} + \zeta\widehat{L}) - \widehat{L} + \widehat{b}. \tag{A.8}$$

Further, bringing back the time subscripts, the two spillovers imply

$$\widehat{a}_t = \alpha_1\widehat{L}_t + \alpha_2\widehat{L}_{t-1} \qquad \text{and} \qquad \widehat{b}_t = \beta_1\widehat{L}_t + \beta_2\widehat{L}_{t-1} + \omega_t. \tag{A.9}$$

Plugging these into Eq. (A.8) proves Eqs. (8) and (9).

For the two-region special case, population conservation gives $\widehat{L}_{1t} = -\widehat{L}_{2t}$ to first order, and subtracting the two regional equations yields Eq. (7) in the log ratios ℓ_t and u_t with the amenity difference $\omega_t = \omega_{1t} - \omega_{2t}$. The same coefficients apply to the coordinate of any nonuniform eigencomponent, because the relations are linear and homogeneous.

All hatted variables and coefficients are dimensionless. When there are no spillovers ($\alpha_1, \alpha_2, \beta_1, \beta_2 = 0$), we have

$$\widehat{u} = (\zeta\mathcal{M}(x) - 1)\,\widehat{L} \qquad \text{and} \qquad \zeta\mathcal{M}(x) - 1 = -\frac{B(x)^2}{\Delta(x)} < 0. \tag{A.10}$$

Thus, the market response alone disperses mobile workers: its competitiveness gain is

smaller than the dilution of income per worker.

A.3 Shape of the multiplier

Direct computation shows

$$\mathcal{M}'(x) = \frac{(2\varepsilon + \gamma\eta)(x - x_*)(\eta x - 1)}{\Delta(x)^2} \qquad \text{with} \qquad x_* = \frac{\gamma + 2\varepsilon\eta}{2\varepsilon + \gamma\eta}. \tag{A.11}$$

Since $\eta x - 1 < 0$ for $x \in (0, 1)$, the shape depends on the location of x_*.

If $\eta = 1$, equivalently $\gamma = 1$, then $x_* = 1$ and $\mathcal{M}$ is strictly increasing. If $\eta < 1$ and $\gamma \geq 2\varepsilon$, then $x_* \geq 1$ and the profile is again increasing. Since $\gamma < 1$, this case requires $\varepsilon < 1/2$. If $\eta < 1$ and $\gamma < 2\varepsilon$, then $0 < x_* < 1$, and $\mathcal{M}$ increases before x_* and decreases afterward. These cases exhaust the allowed domain because $x_* \geq 1$ is equivalent to $(1 - \eta)(\gamma - 2\varepsilon) \geq 0$. Section 3.4 interprets the two profiles and Fig. 1 provides numerical examples.

In the single-peaked case, let $h(x) = \mathcal{M}(x) - \mathcal{M}(x^2)$. Then $h(x_*) > 0$ and $h(\sqrt{x_*}) < 0$. Cross multiplication in $h(x) = 0$ and cancellation of $x(1 - x)$ give

$$2\varepsilon + \gamma\eta = \varepsilon(1 - \eta^2)\left(\frac{1}{1 - \eta x} + \frac{1}{1 - \eta x^2}\right). \tag{A.12}$$

The right side increases strictly on $(0, 1)$, so a unique crossover $\chi^+ \in (x_*, \sqrt{x_*})$ exists. The positive composite $\zeta + \alpha_1 + \alpha_2$ transfers this ordering to G_k^+. If the composite is zero the steady-state forces coincide; if it is negative, the goods-response contribution reverses its ranking.

For the limiting values of trade cost, $\mathcal{M}(0) = \frac{\varepsilon}{1+\varepsilon\zeta}$ and $\lim_{x\uparrow 1} \mathcal{M}(x) = \frac{\gamma}{\zeta}$. When $\gamma = 1$, cancelling the common factor $1 - x$ in (A.7) gives the coefficient in (16), including intermediate inputs. With $\zeta = 1$, this also recovers the no-intermediate-input case. The persistent-agglomeration boundary compares G^+ with Θ^+; the division of spillovers across generations affects the transition roots λ_u and λ_s.

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